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On the Intertemporal Allocation of Time and Money

Martin Browning

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Proefschrift ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op maandag 15 november 1993 te 16.15 uur door

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geboren te Loughborough (United Kingdom).

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To Lisbeth

CONTENTS

CHAPTER 1

INTRODUCTION

INTRODUCTION

Household decisions about how much to save and how much to work are central to many important policy issues. Amongst these we note:

(i) Keynesian policies for short run management of the economy depend critically on how much households change their consumption if governments change disposable incomes by changing income taxes;

(ii) cuts in tax rates on unearned income that are designed to stimulate saving and hence long run rates of growth depend on how responsive household saving is to changes in the interest rate;

(iii) the effects of changes in old age pensions that are designed to decrease poverty amongst old people depend on how responsive savings and the date of retirement are to such changes;

(iv) the design of an efficient unemployment insurance scheme needs a measure of how much labour supply changes consequent on changes in entitlement and levels of benefits;

(v) the availability of maternity leave and subsidised child care may affect the number and timing of children and the levels of education and training that women choose.

The common thread that runs through all these issues is that individuals and households must make decisions that have both short run and long run implications. The most widely used framework within which economists analyse such decisions is the life-cycle model. This framework takes as its starting point the proposition that at any point in time households arrange their affairs so as to maximise a lifetime utility function subject to a lifetime budget constraint using all current information as efficiently as possible. The most important implication of this premise is that agents seek to keep the marginal utility of (discounted) expenditure constant from period to period. This is a very powerful organising principal since, along with supplementary assumptions, it provides predictions about the short run (high frequency/business cycle) and long run (low frequency/life cycle) behaviour of consumption and labour supply (as well as all other life-cycle decisions such as schooling, fertility, occupation choice, portfolio allocations and retirement).

It is critical to recognise that the life-cycle model is only a framework. Without importing more structure we cannot make any predictions that put constraints on what we might observe. Thus all models within this framework (that is, all life-cycle models) can be characterised by the additional assumptions used. For the purposes of this thesis it is necessary to delineate four sets of assumptions that are used. The strongest set of assumptions is the following:

MODEL A. Agents have perfect foresight; preferences can be represented by a utility function that is additive over time; agents have access to perfect capital markets.

There are several ways to relax these assumptions. The most natural replaces the perfect certainty assumption with an assumption of rational expectations and expected utility:

MODEL B. Agents face an uncertain future and have rational expectations; preferences over uncertain outcomes satisfy the postulates of the expected utility model; the VN utility function is additive over time; agents have access to perfect capital markets.

Further weakening can be achieved by dropping the assumption that the VN utility function is additive over time:

MODEL C. Agents face an uncertain future and have rational expectations; preferences over uncertain outcomes satisfy the postulates of the expected utility model; agents have access to perfect capital markets.

Or by dropping the assumption of perfect capital markets:

MODEL D. Agents face an uncertain future and have rational expectations; preferences over uncertain outcomes satisfy the postulates of the expected utility model; the VN utility function is additive over time.

Chapter 2 ("A Profitable Approach to Labor Supply and Commodity Demands Over the Life-Cycle", co-authored with Angus Deaton and Margaret Irish) presents some novel theoretical analysis and some empirical results based on UK data. Throughout the paper it is assumed that the conditions of Model B hold. Under these assumptions we derive various theoretical implications of the model. The derivations of these results are greatly facilitated by the use of an alternative preference representation to the direct utility function. This alternative representation is called the profit function representation. As the paper demonstrates, the use of this representation makes the analysis much easier. Indeed, the principal point of the first half of the paper is to illustrate the use of the profit function representation.

The second half of Chapter 2 presents some results on labour supply and consumption using UK Family Expenditure Survey (FES) data. The empirical analysis of inter-temporal models strictly requires the use of panel data; that is, data that follows the same households over a number of time periods. Unfortunately no such data is available for all consumption items. Many authors have used aggregate time series data but the conditions required to justify the use of such 'representative agent' models seem far too strong to be plausible.

In this paper we propose an alternative way around the absence of panel data on individual households. This alternative uses the FES that is conducted each year in the UK. This survey collects information on the consumption of a comprehensive list of goods by individual households as well as a great deal of other information about the household. Since each household is surveyed only once this seems to rule out any analysis of intertemporal allocation. To illustrate how we overcome this problem consider the derivation of a time series for consumption. We take all agents aged 20 to 24 inclusive in 1970 and take the mean of consumption. We then take all agents aged 21 to 25 inclusive in 1971 and take the mean of consumption and so on until 1976 (for agents aged 26 to 30). We then treat these series of seven cohort means as the consumptions over seven periods by the same agent. Such data is known as synthetic cohort or quasi-panel data. The rationale for this procedure is that although the sample varies over time the population does not (ignoring death and migration) so that the sample means in each period are consistent estimators of the population mean. We then show that the theory applies to the population means.

This procedure is critically different from a representative agent approach in two ways. First, since we have the individual data we can take means of functions of the data. For example, if the theory requires us to work with log consumption then we take logs at the individual level and then take means. If we have only aggregate data we can only take the log of mean consumption. Since the difference between the mean of logs and the log of the mean is an index of inequality the procedure using the aggregate data can only be justified if we assume that changes in inequality are uncorrelated with changes in the mean of consumption. The second difference is that in the representative agent approach the main difference between the population in 1970 and 1971 is that some younger people have 'entered' the population and some older people have 'left'. Even very moderate economic growth will mean that younger people are a good deal better off in lifetime wealth terms than older people. The effects of this on aggregate consumption and savings were the principal focus of the first generation of life-cycle studies (in particular, those of Modigiliani and his co-authors). If the focus of interest is individual behaviour, however, then these changes in the population make inference about individual behaviour from aggregate data impossible unless we impose the most implausible conditions.

The use of quasi-panel data also has an advantage over genuine panel data and that is the absence of attrition. Since each survey is conducted in the same way as the previous one this allows us to build up very long time quasi-panel data sets. For example, recently the 1990 FES became available so that we can now follow the same cohort for 21 years. This allows us to analyse both life-cycle and business-cycle variations in consumption and labour supply. Since the introduction of the quasi-panel method in the first paper in the thesis several authors have investigated the theoretical econometric properties of this estimator and have used such data. It turns out that the method is a generalisation of the Wald 'grouping' estimator that can be used to overcome errors-in-variables problems. As such the quasi-panel method is an Instrumental Variable (IV) estimator and can be justified in that way.

The final part of Chapter 2 uses the theoretical methods introduced in the first part to generate a parameterisation for the model that can be estimated using quasi-panel data. We apply this to male labour supply and consumption. The male labour supply data replicate the stylised facts found by others: for manual and non-manual workers there is a marked synchronisation of hours worked and discounted wages over the life-cycle. This is one of the predictions of the simple life-cycle model. The implied intertemporal substitution elasticity is similar to that found in studies using US data. A closer examination reveals some problems. First, there is clear evidence that the model does adequately account for the year to year changes in hours worked. Thus although the model seems to do well for the life-cycle it does not do so well for the business-cycle. The second problem is that in the hours equation it seems that leisure and consumption are complements whilst in the consumption equation they are substitutes. Formally, this implies a rejection of the symmetry condition which is one of the principal implications of the simple theory model.

Given its central position in the consumption and labour supply literature the simple life-cycle model has been the subject of intensive testing. In Chapters 3 and 4 ("Eating, Drinking, Smoking and Testing the Life-Cycle Hypothesis" and "A Non-Parametric Test of the Life-Cycle Rational Expectations Model", respectively) I present two further sets of tests. The first is more informal than most whereas the second is a good deal more formal and dispenses with the need for specifying a parameterisation for preferences.

In Chapter 3 the alternative to the life-cycle hypothesis is a rather ill-defined model of allocation in which households meet their 'needs' in each period and then use any current income left over (so-called supernumerary income) for the purchase of 'non-essentials' and saving. The usual justification given for such a model is that although all households would like to behave as life-cycle hypothesis agents some are liquidity constrained. Consequently, current consumption is set equal to current income (that is, Model D above). An alternative justification would be that households behave this way and the life-cycle model is simply wrong.

Consider a family that uses such an allocation procedure and that experiences the birth of a child. This has two effects on income and expenditures. First, one of the parents may drop out of the labour force to care for the child. This will lead to a fall in current income. Second, 'needs' rise since children bring with them unavoidable costs. These two effects together work to reduce supernumerary income. The implication is that expenditures on non-essentials like tobacco and alcohol will fall.

This conclusion is reinforced if we also assume that parental preferences change in such a way that they drink and smoke less even if they can still afford it. The predictions for a 'Model B' household are very different. If agents can borrow and lend freely (and preferences over tobacco and alcohol are additively separable from other goods and children) then we would expect to see parents maintaining their previous level of consumption of alcohol and tobacco after the birth of any children. To do this they would either borrow or run down savings.

Thus the informal model and the simple life-cycle model give radically different predictions about the effects of children on the consumption of tobacco and alcohol. In the second half of this paper the quasi-panel data developed in Chapter 2 is used to implement the test. We find that children have no effect on the consumption of tobacco and only a small negative effect on the consumption of alcohol. Although hedged about with considerable reservations this seems to provide some support for the simple version of the life-cycle model in which agents face a perfect capital market and maximise the expected value of an intertemporally additive utility function.

One of the weaknesses of all the tests of the life-cycle model that appear in the literature is that they assume a particular parameterisation for preferences. In Chapter 4 I develop the non-parametric (or revealed preference) implications of the simplest life-cycle model, model A. The specific condition generalises the prediction for the single commodity case: demand is a negative function of discounted price. The generalisation to several goods leads to a set of conditions that can be applied to any time series of purchases, discounted prices, hours of work and discounted wages. The interest rate used to discount prices and wages is, of course, the single interest rate that is assumed to hold in the perfect capital market posited in the assumptions. The set of conditions include Varian's GARP conditions as a subset. This is not surprising since these are required for intra-period 'rationality' whereas the conditions for consistency with the life-cycle model require intra- and inter-temporal 'rationality'. Finally, it is shown that the non-parametric conditions aggregate perfectly; that is, if they hold for each agent then they hold for the aggregate data as well.

These conditions are applied to aggregate time series data from the UK for 1952 to 1985. Although the conditions are rejected for the whole data period they are not rejected for long sub-periods. Moreover, if we allow for 'surprises' in the early 1970's then the non-parametric conditions hold for the whole period. This is of considerable interest since parametric tests of the life-cycle hypothesis on similar data typically reject strongly. The non-rejection of the non-parametric conditions suggests that the rejections reported in the literature may be more a matter of imposing inappropriate parameterisations than of rejecting the hypothesis per se.

Although this looks encouraging for the simplest form of the life-cycle model the specific rejections of the non-parametric conditions over the whole period do point in some specific directions. For example, in each case where the conditions are rejected it is because consumption does not fall as much as predicted by the theory. This suggests a 'ratchet' model of consumption along the lines first discussed by Duesenberry in the late 1940's. Alternatively, we can view the periods for which the non-parametric conditions fail as being those in which agents are liquidity constrained. All of this indicates that the life-cycle model (with some habits or capital market imperfections) does a very good job of characterising the aggregate time series data. What of the principal informal alternative, namely a 'Keynesian' model in which we assume that current total expenditure is an increasing function of current total income and that agents then allocate this total expenditure using stable preferences. In the final section of Chapter 4 it is shown that the data used are also exactly consistent with such an hypothesis. The final conclusion, then, is that the aggregate time series data are virtually worthless for testing the life-cycle model.

The final paper in this thesis (Chapter 5, "A Simple Nonadditive Preference Structure for Models of Household Behavior Over Time") deals with preferences that are not additive over time (that is, it considers Model C above). Introspection and much of the formal evidence on the life-cycle model suggests that preferences are not additive over time. Some goods are habit-forming and others are satiating. These considerations are obvious and lead to a plethora of non-additive models in the early 1950's at the same time that the empirical analysis of consumption decisions itself began. Gradually, however, the additive model came to dominate. The reason is not that it gave as good a fit to the data as non-additive models but that it is an order of magnitude easier to handle. The reason is the following. If present preferences depend on past choices then when modeling current demands we have to take account of past purchases. This is not too difficult. However, if the past affects the present then the present will affect the future so that we also need to take into account future prices. To illustrate, consider tobacco. This is habit forming and someone who smokes now will be sensitive to projected future changes in the relative price of tobacco. If the government announces that it is going to increase the price of tobacco in the future then forward looking agents will cut back now. It is this twin dependence of current demands on past quantities and future prices that make the structural modeling of non-additive preferences so difficult.

A variety of models have been suggested to deal with inter-temporal non-separabilities. From an econometric point of view the simplest model would have current demands depending on one period lagged quantities and one period lead prices. This cuts down on the number of leads and lags and has the lead variable being exogenous. In Chapter 5 it is shown that unfortunately there is no way such a model can be reconciled with a utility framework (unless, of course, preferences are additive). Indeed, it can be shown that if we wish to include only one period lagged quantities then current demands will depend on all future prices. Of course, we could overcome this by making ad hoc assumptions about the future path of prices but this runs against the modern tide of minimising such ad hoc assumptions about future variables.

Conversely, it is shown that if we include only one period lead prices then we need to include all lagged quantities. Once again, this can be overcome by making ad hoc assumptions but this is is also unpalatable.

The approach in Chapter 5 is to derive the simplest possible generalisation of additivity where 'simple' refers to the data needs. A preference structure is developed that allows current demands to depend on current prices and one period lead and lag prices. I characterise the preferences that satisfy this condition and term it a Simple Non-Additive Preference (SNAP) structure. The first section of Chapter 5 develops some of the implications of a SNAP structure in a world of perfect certainty. It is shown that it includes the standard durables case as a special case. The second section shows how to extend the framework to allow for uncertainty. The most important result there is that if SNAP is be empirically useful then either we have to restrict preferences significantly or we must assume that agents have point expectations about future prices.

In the following two sections the SNAP model is applied to some UK aggregate time series data. The particular parameterisation chosen includes the Almost Ideal demand model of Deaton and Muellbauer (1970) as a special case. The SNAP generalisation introduces only a few more parameters (strictly, one less than the number of goods being modeled). When applied to the UK data we find that inter-temporal additivity is rejected. Significantly, however, the inter-temporal dependencies are mostly 'concentrated' on durables which is hardly a surprise. In fact, it seems all other goods (with the possible exception of fuel which displays some problems anyway) are intertemporally 'separable' (in a sense made exact in the paper). Thus it seems that most of the apparent non-separabilities seen in past demand studies can be attributed to the exclusion of durables from such systems. Since the demands for other goods are not separable form durables this lead to an apparent need to allow for inter-temporal non-separabilities in, say, food or services.

CHAPTER 2

A PROFITABLE APPROACH TO LABOR SUPPLY AND COMMODITY DEMAND OVER THE LIFE-CYCLE

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A PROFITABLE APPROACH TO LABOR SUPPLY AND COMMODITY DEMANDS OVER THE LIFE-CYCLE

BY MARTIN BROWNING, ANGUS DEATON, AND MARGARET IRISH[1]

The paper presents a general theoretical framework for the analysis of integrated life-cycle models of consumption and family labor supply under uncertainty. Profit functions are used to represent intertemporally additive preferences and to yield convenient characterizations of "constant marginal utility of wealth" or "Frisch" demand functions. Conditions on preferences are derived that allow additive fixed-effect specifications for the Frisch demands. Data from the British Family Expenditure Surveys from 1970–77 are used to derive panel-like information on male labor supply and consumption for several age cohorts over time. These data reproduce standard life-cycle patterns of hours and wages, but more detailed analysis shows that the theory is incapable of offering a satisfactory common explanation of the behavior of hours and wages over both the business cycle and the life cycle. Similarly, although the theory can explain the life-cycle behavior of hours and consumption separately, the same model cannot explain both, essentially because of a failure in symmetry.

INTRODUCTION

OUR OBJECT IN THIS PAPER is to provide a general theoretical framework for the empirical analysis of integrated life-cycle models of consumption and family labor supply. We also use data from the 1970 to 1977 Family Expenditure Surveys of the United Kingdom to estimate life-cycle models of male hours and household consumption. The way in which we combine time-series and cross-section data allows a simultaneous analysis of behavior over both business and life cycles. We find that, although much of our evidence is broadly interpretable in terms of life-cycle theory, the theory is *not* capable of offering a *common* explanation of the business cycle and the life cycle, nor of consumption and hours, even though each can be explained in isolation.

Previous British studies of labor supply have not taken a life-cycle view. The studies referenced in the survey by Greenhalgh and Mayhew [**24**] use either aggregate data from time series and industrial cross-sections or micro data from household surveys and are based on the standard static model of labor supply. Typically, such studies find backward sloping supply curves for prime-age males together with small negative effects on labor supply of assets or unearned income or proxies for them; it is not always clear that the implied substitution elasticity is positive as required by theory. Indeed, in studies using the Family Expenditure Survey, unconstrained regressions tend to produce backward sloping supply

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curves together with zero or positive income effects, clearly contradicting theoretical presupposition; see Atkinson and Stern [**6**] and Deaton [**12**]. Of course, there are severe difficulties in obtaining good data on unearned income in any household survey and there are serious conceptual problems in applying the static model to such data as exist. In a life-cycle context, assets and asset income are not exogenous vaiables but evolve systematically with the labor supply and spending decisions of the household. It is easy to imagine the same household at different points in the life cycle first working long hours with low assets, then working long hours with high assets and yet later working short or no hours with low assets. The only truly exogenous asset variables are inherited assets on the one hand, and asset "surprises" on the other; these are not recorded in standard household surveys. There are similar problems interpreting wage responses in the standard static model. It is reasonable to suppose that a fully anticipated wage increase will have different effects from an unanticipated change since the latter changes the individual's perception of life-time resources while the former does not. Similarly, most households are not surprised when their children attain an age at which their financial demands on their parents becomes very large, so that such events must begin to influence labor supply and savings plans long before they occur. All these phenomena require an explicitly life-cycle perspective as well as a proper integration of labor supply and consumption behavior.

These issues have been recognized in the American literature for some time, although the static model is still the dominant framework of analysis. Mincer's [**40**] model of female participation is explicitly set in a life-cycle background, but modern developments in life-cycle labor supply begin with Heckman's [**28**] Princeton doctoral thesis; see Heckman [**29** and **30**], and with Ghez and Becker [**18**]. Ghez and Becker make much of the important distinction between anticipated wage changes *along* the life-cycle wage profile and unanticipated changes which shift the profile itself. Smith's [**45**] paper is in this tradition but, like Ghez and Becker's work, the analysis is hampered by lack of panel data, so that averages of workers at specified ages (synthetic cohorts) are assumed to represent behavior along a single profile for all workers. Heckman's analysis provides the basis for an appropriate theoretical treatment by showing that the supply functions required for the analysis are neither those holding wealth constant nor those holding utility constant, but those that hold *marginal* utility constant. This is the starting point for much of the analysis in this paper. Marginal utility constant demand functions also turn out to be central in the analysis of intertemporal choice under uncertainty and provide a bridge between the labor supply literature and the "rational expectations" consumption function models of Bewley [**9**] and of Hall [**25**] which trace back ultimately to the intertemporal arbitrage conditions of the finance literature. The final important development is due to MaCurdy (see Heckman [**31**], Heckman and MaCurdy [**32**], and MaCurdy [**37**]). This is the realization that, at least in certain specifications, the essentially unobservable marginal utility is constant over the lifetime of the consumer and so, given panel data, can be treated as a fixed effect in the econometric analysis.

In this paper, our first aim is to develop the full theoretical basis for marginal utility constant demand functions and to relate them to standard concepts in the theory of consumer behavior. In particular, we discuss the relevant duality theory, an appreciation of which gives great advantages, not only in understanding, but also in ease of selecting functional forms and of relating empirical observations to the forms of preferences thereby implied. The key to the analysis turns out to be the life-cycle *profit function*, first discussed (in the context of the Rotterdam model) by Gorman [**22**]. The concept is identical to that used in production theory (see, e.g., McFadden [**38**]), and in the same way it has demand functions as partial derivatives, in this case the marginal utility constant demands, or as we call them here, the *Frisch* demands. In the life-cycle context with uncertainty, the Frisch demands neatly separate anticipated from unanticipated effects, not only of wages on labor supply and participation, but also of commodity prices and demographic structure on both labor supplies and commodity demands. The close link between demands on the one hand and profit and cost functions on the other allows us to use standard techniques from demand analysis to incorporate in a systematic way the presence of children and to predict the effects of the demographic life-cycle on male and female labor supply and household commodity demands.

The first part of the paper takes up these issues in turn. Section 1 is a general discussion of additive preferences and of the characterization of consumer preferences by profit functions; only a summary is given and a fuller analysis can be found in Browning [**10**]. Section 2 applies the results of Section 1 to the life-cycle, derives life-cycle and age-specific profit functions and gives the general results governing labor supplies, participation, and commodity demands in terms of the Frisch demands. Section 3 introduces uncertainty into the intertemporal choice problem and links the Frisch demands to the literature on the consumption function, particularly to Bewley [**9**] and Hall [**25**]. Section 4 contains some simple exercises in comparative statics and dynamics designed to illustrate the power of the model to generate testable hypotheses and to tell "stories" about home economics. For example, under plausible assumptions, anticipated increases in men's wages cause their wives to work *more*. Similarly, the birth of an additional child may cause the husbands of nonparticipating wives to work longer hours but have no effect on the hours of those men whose wives continue to participate in the labor market after the increase in family size.

The second part of the paper is concerned with the selection of appropriate functional forms and with empirical implementation on Family Expenditure Survey data. Section 5 takes up MaCurdy's suggestion of treating the marginal utility as an unobservable fixed effect. We derive the general restrictions on preferences that allow such a formulation and propose from within the class a set of flexible functional forms that permit the testing of a number of important restrictions on behavior. These include separability of husband's and/or wife's leisure from each other and from goods, in addition to the usual symmetry restrictions of demand theory. Section 6 discusses an important device that allows us to use the Family Expenditure Survey to generate what are effectively panel data. In the United Kingdom, we do not have genuine panel data on incomes, hours, and commodity demands. However, we are unusually fortunate in having a *continuous* household survey, the FES, that generates random samples of the population in every year. With complete enumeration, i.e., with census data, we could follow cohorts through time. With a continuous random sample, we can follow cohort *means* through time to the extent that sample means are good estimates of population means. Although we cannot follow individual households through time, we can look at the average behavior of 25 year olds one year, of 26 year olds the next, and so on, thus following actual, not synthetic cohorts. Such data even have certain advantages over panel data, notably the preservation of randomness through the absence of attrition. Section 6 explains exactly how the data were extracted from the FES; the means used in the study although based on nearly 50,000 original observations, make up a relatively small data set which is listed in the Appendix. Section 7 contains the results of an application of our model to both male labor supply and to aggregate household expenditures. The hours data replicate the stylized facts found by others, that for manual and nonmanual workers there is a marked synchronization over the life-cycle between hours worked and discounted wage rates with workers working longest hours when it is most profitable to do so. On such evidence alone, the elasticity of weekly hours to anticipated wage changes is around 0.15, a figure in accord with MaCurdy's [**37**] estimate for the United States. However, more detailed analysis casts considerable doubt on the simple life-cycle explanation. In particular, sensible positive responses between hours and wages are only consistently obtained when year to year changes are separately allowed for by dummy variables. Secondly, the characteristic hump-shaped patterns of both hours and real consumption, though explicable in terms of life-cycle wage variation, can be explained as well as or better by other factors, particularly by the demographic composition of the household. Hence, life-cycle patterns could be interpreted as the response of credit-constrained consumers to the variation in needs accompanying the birth, growth, and departure of children. Finally, a life-cycle interpretation of consumption expenditures requires that consumption and leisure be substitutes, while our estimates of male labor supply imply almost as strongly that leisure and consumption be complements. Overall then, we find a considerable amount of evidence that is contrary to the simple life cycle story.

PART ONE: THEORY

1. ADDITIVE PREFERENCES AND PROFIT FUNCTIONS

Consider first a quite general model of consumer choice with additive preferences. We write this

$$\max_{q_i} \sum_{i=1}^{n} \nu_i(q_i) \quad \text{subject to } p \cdot q = x \tag{1.1}$$

where q_i is the quantity purchased of each of n goods, p_i is the corresponding price, and x is the predetermined expenditure total. If we assume convexity of preferences, all but one of the subutilities must be concave (see Yaari [**46**]); we assume further that *all* the subutility functions ν_i are strictly concave and twice differentiable. We *temporarily* make the assumption of internal solutions; these are characterized by first-order conditions

(1.2) $\quad \nu_i'(q_i) = \lambda p_i = p_i/r$

where λ is a Lagrangian multiplier representing the marginal utility of x, given that utility is normalized by taking the explicitly additive form. The quantity r, which plays a key role in what follows, is defined as the reciprocal of λ, i.e., as the marginal cost of utility or better, as the *price of utility*. Since each $\nu_i(\quad)$ is strictly concave, $\nu_i'(\quad)$ is monotone decreasing so that (1.2) can be inverted to give

(1.3) $\quad q_i = f_i(p_i/r)$

for monotone decreasing functions $f_i(\quad)$.

Ragnar Frisch [**16**] was one of the first writers to systematically use additive preferences to measure the marginal utility of money, and following Browning [**10**], we refer to the demand functions (1.3) as *Frisch demands.* Under the additivity assumption, Frisch demands characterize quantities purchased in terms of a single quantity, the ratio of the commodity price to the price of utility. Such demands should be distinguished from the usual uncompensated or Marshallian demands that relate quantities to prices and total outlay, as well as from the compensated or Hicksian demands that relate quantities to prices and utility. The Frisch demands can be transformed into Marshallian demands by solving for r in terms of p and x by applying the budget constraint to (1.3), i.e., from

(1.4) $\quad \sum p_i f_i(p_i/r) = x,$

or into Hicksian demands by expressing r in terms of u and p through

(1.5) $\quad \sum p_i f_i(p_i/r) = c(u, p)$

where $c(u, p)$ is the cost or expenditure function corresponding to the original preferences. The conceptual experiment corresponding to a Frisch demand is one in which consumers are money compensated for a price change until their price of utility returns to its original value. But a more useful and natural interpretation will appear in the life-cycle context.

The analysis clearly extends to "block" additivity or strong separability where the subutility functions are defined over groups of goods rather than single goods. Problem (1.1) becomes

(1.6) $\quad \max u = \sum_G \nu_G(q_G) \quad \text{subject to} \quad \sum p_G \cdot q_G = x$

where q_G and p_G represent price and quantity vectors for group G. The solution follows the same lines and the Frisch demands are, for good i in group G,

(1.7) $\quad q_{Gi} = f_{Gi}(p_G/r)$

so that demands in the group depend only on prices in the group relative to the price of utility.

Utility is an "output" for the consumer so that Frisch demands can be thought of as relating optimal inputs to the prices of output and the inputs. In production theory such demands arise in the analysis of a profit maximizing firm and this familiar apparatus turns out to be conveniently adaptable to the consumer context. Hence, define the consumer's profit function as the maximum profit attainable from selling utility (to him or herself) at a price r, subject to the technology of utility production, i.e., the utility function, and the prices of the inputs. For a general utility function $\nu(q)$, we write this

$$\pi(r, p)=\max_{u,q}\{ru-p\cdot q\,;\,u=\nu(q)\}. \tag{1.8}$$

The existence of $\pi(r, p)$ for all $p\geq 0$ requires that $\nu(q)$ be strictly concave and co-finite; see Rockafellar [**44**] and Lau [**36**]. Note that, by its definition, $\pi(r, p)$ is convex and linear homogenous in (r, p), increasing in r and decreasing in p (see McFadden [**38**]). An alternative derivation of the profit function that is frequently useful is

$$\pi(r, p)=\max_{u}\{ru-c(u, p)\}. \tag{1.9}$$

In many applications it is easier and more convenient to place structure on the cost function so that (1.9) is useful in deriving the corresponding structure for the profit function.

At a formal level, $\pi(r, p)$ is (minus) the concave conjugate of $r\nu(q)$ with respect to q and the convex conjugate of $c(u, p)$ with respect to u; see Rockafellar [**44**] for a discussion of conjugacy. Since the original functions are the conjugates of their own conjugates, utility and cost functions can be retrieved from the profit function using the two identities

$$c(u, p)=\max_{r}\{ru-\pi(p, r)\}, \tag{1.10}$$

$$r\nu(q)=\min_{p}\{p\cdot q+\pi(p, r)\}. \tag{1.11}$$

At an economic level, the profit function represents consumer preferences as a function of the price of utility and the prices of goods just as, for example, the cost function represents preferences in terms of utility and goods' prices. For the latter, demands can be obtained by differentiation, and the same is true for the profit function. From (1.9), we have immediately

$$-\frac{\partial\pi}{\partial p_i}=\frac{\partial c}{\partial p_i}=q_i=f_i(r, p), \tag{1.12}$$

$$\frac{\partial\pi}{\partial r}=u=f_0(r, p). \tag{1.13}$$

Hence, just as the cost function is the potential function for the Hicksian demands, the profit function is the potential function for the Frisch demands. These relationships allow us to derive the general properties of Frisch demands as well as providing the link between preferences on the one hand and the empirical analysis on the other.

We conclude this section by noting the properties of Frisch demands that we shall require in the subsequent analysis.

(A) Frisch demands are zero degree homogenous in r and p. This follows immediately from the linear homogeneity of $\pi(r, p)$ and the derivative property (1.12). See also (1.3) and (1.7).

(B) Frisch demands have symmetric derivatives, i.e.,

$$\partial f_i / \partial p_j = \partial f_j / \partial p_i. \tag{1.14}$$

The derivative matrix is simply minus the Hessian of the profit function so that symmetry follows by Young's theorem. This symmetry is similar to but not the same as Slutsky symmetry. The relationship between Frisch and Slutsky responses is derived by writing

$$q_i = c_i(\pi_0(r, p), p) \tag{1.15}$$

where c_i is the derivative of $c(u, p)$ with respect to p_i and π_0 that of $\pi(r, p)$ with respect to r. Differentiation with respect to p_j and rearrangement gives the relationship between utility compensated and utility-price compensated derivatives as

$$s_{ij} = f_{ij} + \frac{\partial q_i}{\partial x} \cdot \frac{\partial q_j}{\partial x} \cdot x\breve{\omega}^{-1} \tag{1.16}$$

where s_{ij} is Slutsky substitution, f_{ij} is Frisch substitution, and

$$\breve{\omega} = \partial \ln r / \partial \ln x \tag{1.17}$$

is Frisch's [**17**] income flexibility of the marginal utility of money. Equation (1.16) decomposes substitution effects into "specific" substitution effects (the f_{ij}) and "general" substitution effects (terms coined by Houthakker [**33**]); see also MaCurdy [**37**]. Note that conjugate functions have Hessians which are mutual inverses so that the Frisch substitution matrix is proportional to the inverse of the Hessian of the utility function; see the expressions for f_{ij} in Barten's [7] "fundamental matrix equation of demand theory."

(C) Frisch demands slope downwards. By its defintion $\pi(r, p)$ is convex so that its Hessian, where it exists, is positive semi-definite. Since the (f_{ij}) matrix is the negative of this Hessian, it is negative semi-definite. Hence

$$\partial f_i / \partial p_i \leq 0 \tag{1.18}$$

or more generally, for price vectors p^1 and p^0,

$$\{f(r, p^1) - f(r, p^0)\} \cdot \{p^1 - p^0\} \leq 0. \tag{1.19}$$

(D) Additive (or block additive) utility is equivalent to additive (or block additive) profits. It is intuitively clear that additive utility functions allow decentralization of utility production provided each production unit produces output (utility) at the same price, in this case r. Formally, take the strongly separable case (1.6). Then

$$(1.20)\qquad \pi(r,p)=\max_{u,q}\{ru-\sum p_G\cdot q_G;\ u=\sum \nu_G(q_G)\}$$
$$=\max_{u_G,q_G}\{r\sum u_G-\sum p_G\cdot q_G;\ u_G=\nu_G(q_G)\}$$
$$=\sum_G \max_{u_G,q_G}\{ru_G-p_G\cdot q_G;\ u_G=\nu_G(q_G)\}$$
$$=\sum_G \pi_G(r,p_G).$$

Hence, the overall profit function is the sum of the individual profit functions corresponding to each subutility function. Overall profits are the sum of branch profits; input prices are branch specific but the output price is the same for all branches and provides the (only) link between them.

Differentiation of (1.20) yields branch Frisch demands which are a function of group prices and the price of utility alone, i.e., of the form (1.7). The Hessian of (1.20) is block diagonal as is the Hessian of a strongly separable utility function. The effect of additivity is thus to set to zero all cross-branch specific substitution effects. In the next section we shall see how useful this is in the context of intertemporal choice.

2. THE LIFE CYCLE, PROFITS, AND DEMANDS

We begin with the case of perfect certainty and assume that family life-cycle preferences can be represented by the utility function

$$(2.1)\qquad u=\sum_0^L \nu_t(l_{1t}, l_{2t}, q_t)$$

where $t=0,\ldots,L$ indexes age, l_{1t} is leisure of the type 1 worker (husband), l_{2t} is leisure of the type 2 worker (wife), and q_t is a vector of household consumption levels. The period subutility functions $\nu_t(\quad)$ are indexed on age t; this could reflect intertemporal discounting of utility (if such a phenomenon is thought to be sensible), but more importantly the variation with t recognizes the modifying role played by the presence of children and their changing demands over the family life cycle. The intertemporal strong separability that is assumed by (2.1) is a crucial element in all of our analysis and only a limited number of our results hold without it. The fact that additivity is an almost universal assumption in work on intertemporal choice does not suggest that it is innocuous.

For the moment, assume that utility is maximized under perfect certainty in which case the life-cycle budget constraint, discounted back to age 0, can be written

$$(2.2)\qquad \sum_0^L \hat{p}_t\cdot q_t+\sum_0^L \hat{w}_{1t}l_{1t}+\sum_0^L \hat{w}_{2t}l_{2t}=A_0+\sum_0^L(\hat{w}_{1t}T_{1t}+\hat{w}_{2t}T_{2T}).$$

In this expression w_{1t} and w_{2t} are the wages of husband and wife at age t, A_0 is the present discounted value at 0 of nonhuman assets, and T_{1t} and T_{2t} are the age t time endowments for the husband and wife respectively. A caret over a price or wage indicates that the variable is discounted to its present value; for age t, the discount factor is the product of all single period discount factors from 0 to t. Explicit formulae are considered later.

The problem we are concerned with is the maximization of (2.1) subject to (2.2) with l_{1t}, l_{2t} and q_t as instruments. We implicitly assume that the household has already made its fertility and human capital plans; however this has been done, the optimization problem is still correct although the wage rates and the numbers and ages of children cannot be taken as parametric. Allowance for these effects must therefore be made as necessary in the econometric work.

The intertemporal additivity assumption allows decentralization over time (age). Each period of life is regarded as the site for an independent utility factory and lifetime utility is the sum of all the individual plant outputs. The link between periods is the discounted price of lifetime utility, i.e., the reciprocal of the marginal utility of lifetime wealth (or full income). Define then the age t profit function by

$$\text{(2.3)} \qquad \pi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = \max_{u, l_{1t}, l_{2t}, q_t} \{ru + \hat{w}_{1t}(T_{1t} - l_{1t}) + \hat{w}_{2t}(T_{2t} - l_{2t}) - \hat{p}_t \cdot q_t;\ v_t(l_{1t}, l_{2t}, q_t) = u\}.$$

Profits accrue from sales of utility and sales of the two kinds of market labor offset by the costs of inputs. By the arguments of Section 1, we then have

$$\text{(2.4)} \qquad \frac{\partial \pi_t}{\partial \hat{w}_{1t}} = h_{1t} = f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

$$\text{(2.5)} \qquad \frac{\partial \pi_t}{\partial \hat{w}_{2t}} = h_{2t} = f_{2t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

$$\text{(2.6)} \qquad -\frac{\partial \pi_t}{\partial \hat{p}_{ti}} = q_{it} = \phi_{it}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t),$$

where h_{1t} and h_{2t} are the hours of market work supplied by each type of worker, f_{1t} and f_{2t} are the Frisch supply functions for labor, and ϕ_{it}, $i = 1, \ldots, n$ are the Frisch demand functions for commodities.

These equations show immediately the benefit of working with Frisch demands. As emphasized in the work of Heckman and of MaCurdy, labor supplies and commodity demands are a function of immediately observable within period variables such as prices and wages while all the variables from outside the period, many of which are unobservable (future prices, wage rates, and so on), are represented by a single "sufficient statistic" r which, at least under perfect certainty, does not vary from period to period. Such equations are the perfect answer to the difficulties facing the econometrician who attempts to estimate life-cycle models. However, there is still a number of difficulties to be faced both in this section and the next.

Note first that the formulae (2.4) to (2.6) assume internal solutions for the labor supplies and commodity demands. The constraints $q_{ti} > 0$ and $T_{it} > h_{it} > 0$

have been ignored although in practice some are likely to be binding. We illustrate for the case of nonparticipation by the wife, i.e., for $h_{2t}=0$. The same principles apply to the other inequality constraints (although large numbers of regimes are hard to handle). Note first that if h_{1t}, h_{2t} and q_{ti} as given by (2.4) to (2.6) are all nonnegative, then these values are optimal and there is nothing more to be said. Consider then the case where (2.5) yields a value of $f_{2t}<0$. Since the Frisch demands are the inverses of the first order conditions, $f_{2t}<0$ implies that at zero hours, a decrease in home time is worth more than the wage earned, so that $h_{2t}=0$ is optimal. Hence, as might be expected, positive Frisch labor supply functions correspond to positive hours worked while negative Frisch supplies indicate nonparticipation. This is the familiar Tobit specification for censored distributions. Even so, when the wife does not participate, the husband's labor supply and household commodity demands will generally be different since the optimization must allow for the effective ration at $h_{2t}=0$. The analysis of this situation requires a *restricted* profit function for age t in which h_{2t} is set to zero. The details are essentially identical to those given for restricted cost functions in Neary and Roberts [**41**] and Deaton [**11**] and the solution is characterized by the following equations:

$$h_{1t}=f_{1t}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t), \tag{2.7}$$

$$0=f_{2t}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t), \tag{2.8}$$

$$q_{it}=\phi_{it}(r, \hat{w}_{1t}, \hat{w}_t^*, \hat{p}_t). \tag{2.9}$$

In equation (2.8) $\hat{w}_t^*$ is defined as the wage rate that causes the wife to wish to work zero hours at age t; it is her reservation wage (sometimes shadow or virtual wage). Note that (2.8) yields a unique solution for $\hat{w}_t^*$ since $f_{2t}(\quad)$ is monotone increasing in $\hat{w}_{2t}$. Husband's labor supply and household consumption demands have the same functional form as before with the wife's reservation wage replacing the actual wage. This has important implications. Consider, for example, a variable that in (2.4)-(2.6) affects only wife's labor supply, for example the presence or absence of an infant. Once the wife ceases to participate, (2.8) becomes relevant and changes in the variable will alter $\hat{w}_t^*$ and hence the husband's labor supply as well as the commodity demands.

From an econometric point of view (2.4)-(2.6) and (2.7)-(2.9) should be regarded as two systems of equations with endogenous switching determined by whether or not f_{2t} is negative. Note that the presence of nonparticipation at some point in the life cycle does not affect the constancy of r; the price of utility is a datum for the life of the family and it is not altered as behavior switches from one set of equations to the other.

3. THE TREATMENT OF UNCERTAINTY

We begin by rewriting the life-cycle Frisch demands under certainty. We take the male labor supply equation (2.4) as representative:

$$h_{1t}=f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t); \tag{3.1}$$

omitting the others merely saves space. The wage rates and prices in this equation are discounted back to age 0; hence, taking p_{it} to illustrate:

$$(3.2) \qquad \hat{p}_{it} = p_{it}\delta(t, 0) = p_{it} \prod_{0}^{t-1} (1 + i_\tau)^{-1},$$

where $\delta(t, 0)$ is the discount factor to be applied to period t in period 0 and i_τ is the *nominal* rate of interest linking period τ with period $\tau+1$. Since (3.1) is zero degree homogeneous, we can divide through by $\delta(t, 0)$ to give

$$(3.3) \qquad h_{1t} = f_{1t}(r_t, w_{1t}, w_{2t}, p_t)$$

where $r_t = r/\delta(t, 0)$ is the price of utility in period t, or, more precisely, the price of lifetime utility in terms of period t's money. Because of the discounting, r_t, unlike r, is not constant with age but, by definition, evolves according to

$$(3.4) \qquad r_{t+1} = r_t(1 + i_t).$$

This equation guarantees that $r_t\delta(t, 0)$, the discounted price of lifetime utility, is the same at all ages (periods). Equations (3.3) and (3.4) taken together are precisely equivalent to the original (3.1) and provide a convenient characterization using only currently dated magnitudes.

Choice under uncertainty is here characterized by expected utility maximization with continuous replanning. Labor supplies and commodity demands at t are thus chosen to maximize

$$(3.5) \qquad \nu_t(l_{1t}, l_{2t}, q_t) + E_t \sum_{t+1}^{L} \nu_k(l_{1k}, l_{2k}, q_k)$$

where $E_t(\cdot)$ is the expectations operator conditional on information available at time t. By taking this form, we assume that the same explicitly additive form of utility can be used to characterize both intertemporal separability and the additivity over states that is implied by the conditional preference axiom of choice under uncertainty. There is no automatic guarantee that this should be so; nevertheless we believe that the characterization embodies the most reasonable interpretation of intertemporal additivity under uncertainty. To summarize the argument, start from the sure-thing principle *without* intertemporal separability, so that preferences can be represented by an additive function $\sum \pi_s G(q_s^1, q_s^2, \ldots, q_s^L)$, with π_s the probability of state s, and q_s^t (temporarily) representing the vector of consumptions and leisures in period t and state s. The crucial question is exactly what is meant by intertemporal additivity in this context. A *minimum* requirement is that, under certainty, utility be additive, or equivalently, that within any given state, utility be additive. In terms of basic preference orderings, we require that for *given* s, the conditional ordering over any pair $\{q_s^t, q_s^{t'}\}$ must be independent of τ for $\tau \neq t, t'$. This implies that utility can be represented by the form $\sum \pi_s F\{\sum \nu_t(q_s^t)\}$. One could reasonably stop at this point, but we feel it is more appropriate to require that preferences be *simultaneously* additive over periods and states. Without restriction on $F\{\ \}$, conditional orderings over, for example, $\{q_s^t, q_{s'}^t\}$ are not independent of q_s^τ and $q_{s'}^\tau$ for *all* $\tau \neq t$. In consequence, my preferences over picnic/sun versus movie/rain tomorrow are not independent of all my future consumption levels in future states that encompass sun or rain tomorrow. We prefer not to characterize such preferences as being intertemporally additive and instead to require simultaneous additivity, so that the conditional ordering of $\{q_s^t, q_{s'}^{t'}\}$ be independent of q_{σ}^{τ} for all $\tau \neq t$, t' and $\sigma \neq s$, s'. By the application of Gorman's [**20**] overlapping separability theorem, such simultaneity requires that F be at most an affine transformation and that preferences can be represented by the doubly additive form, as in (3.5). Some of the further complications introduced by recognizing the sequential resolution of uncertainty over time are discussed by Gorman [**23**].

The maximization of (3.5) subject to the life-cycle budget constraint requires the usual recursive substitution from L back to t that is characteristic of stochastic dynamic optimization; for a good exposition see, for example, Epstein [**15**]. Note first that the life-cycle utility function (3.5) still has a two period intertemporally additive structure between "now," period t, and the "expected future," from $t+1$ to L. Consequently, all the previous apparatus of Frisch demands goes through, i.e., we can write exactly as before

$$h_{1t} = f_{1t}(r_t, w_{1t}, w_{2t}, p_t) \tag{3.6}$$

for the period t male labor supply, conditional on r_t, the price of expected lifetime utility *as perceived at* t. The only difference between certainty and uncertainty is the process controlling the evolution of r_t, and it is this that is derived from the dynamic optimization. To simplify notation, define period τ "full" expenditure as

$$x_\tau = w_{1\tau} l_{2\tau} + w_{2\tau} l_{2\tau} + p_\tau \cdot q_\tau, \tag{3.7}$$

let $\psi_\tau(x_\tau, w_{1\tau}, w_{2\tau}, p_\tau)$ be the period τ indirect subutility function, $\omega_\tau = w_{1\tau}T_{1\tau} + w_{2\tau}T_{2\tau}$ period τ's endowment, and A_τ assets at the beginning of τ. The evolution of assets is given by

$$A_{\tau+1} = (A_\tau + \omega_\tau - x_\tau)(1 + i_\tau). \tag{3.8}$$

Let $\psi_\tau^*(A_\tau)$ be the sum of current and expected future utilities as perceived at age τ given assets A_τ inherited from age $\tau - 1$. At the end of life at age L we have

$$\psi_L^*(A_L) = \psi_L(A_L + \omega_L, w_{1L}, w_{2L}, p_L), \tag{3.9}$$

while for any other $\tau < L$, optimization over period τ full expenditure gives

$$\psi_\tau^*(A_\tau) = \max_x \{\psi_\tau(x, w_{1\tau}, w_{2\tau}, p_\tau) + E_\tau[\psi_{\tau+1}^*((A_\tau + \omega_\tau - x)(1 + i_\tau))]\}. \tag{3.10}$$

In particular, (3.10) holds for the present, $\tau = t$, so that the first-order condition is

$$\partial\psi_t/\partial x = E_t\{(1 + i_t)\, \partial\psi_{t+1}^*/\partial x\}. \tag{3.11}$$

But $\partial\psi_t/\partial x$ is simply the marginal (lifetime) utility of period t's money, or the reciprocal of the undiscounted price of utility. Hence, (3.11) becomes

$$(3.12) \qquad E_t\{(1+i_t)r_t/r_{t+1}\}=1.$$

This equation is the counterpart of (3.4) in the certainty case; under certainty, (3.12) holds without the expectation. Thus, provided we work with Frisch demand functions, the incorporation of uncertainty is straightforward. The Frisch demands, in undiscounted form, are unchanged, but the price of utility follows a stochastic rather than a deterministic process. Equation (3.11) or (3.12) is the standard stochastic Euler equation of intertemporal equilibrium, familiar from the theory of stock-market prices or from optimal accumulation under uncertainty. It is also the basis for Bewley's [**9**] and Hall's [**25**] versions of the permanent income model.

Recent work by Hansen and Singleton [**27**] has provided an econometric procedure for direct estimation of the Euler equation together with the other first-order conditions (i.e., the Frisch demands). The procedure has also been used by Mankiw, Rotemberg, and Summers [**39**] to study aggregate consumption and labor supply in a paper with similar aims to the current one. Our own approach is to write (3.12) as

$$(3.13) \qquad (1+i_t)/r_{t+1}=1/r_t+\varepsilon_{t+1}, \quad E_t(\varepsilon_{t+1})=0,$$

and then to take logarithms and approximate to give

$$(3.14) \qquad \ln r_{t+1} \simeq \ln r_t + \ln(1+i_t)+\eta_{t+1}$$

with $\eta_{t+1}=-r_t\varepsilon_{t+1}$ and $E_t(\eta_{t+1})=0$. This technique, unlike Hansen and Singleton's, requires an approximation that clearly removes some of the theoretical sharpness deriving from the rational expectations modelling. The compensating advantage is that the simple structure of the Frisch demands under certainty is preserved, and that the model has the certainty model as a special case when $\eta_{t+1}=0$. In consequence, we shall be able to deal below with the uncertainty case by straightforward differencing and instrumentation of the standard regressions that will represent the model under certainty.

We now have a clear interpretation of life-cycle Frisch demands. With perfect foresight and no uncertainty, consumers track along their predetermined life-cycle trajectories of labor supply and consumption demand. With uncertainty, new information is constantly coming to hand. If the new information leaves r unchanged, or permits (3.12) to hold exactly (not just in expectation), it is as if there had been no new information, and the consumer continues along the predetermined path. The price and wage derivatives of the Frisch demands are derivatives with the utility price constant and so are the derivatives of this predetermined path. It is perhaps not misleading to call them derivatives with respect to *anticipated* changes, although in this context fulfilment of expectations is defined by $r_{t+1}=(1+i_t)r_t$. The great advantage of the Frisch demands is that they separate out the effects of movements along the path (which occur with or without perfect foresight) from those movements of the path itself caused by new information.

There is another important issue that has not been sufficiently emphasized in the literature and which has been brought to our attention by Larry Epstein. Consider equation (3.1), the male labor supply equation under *certainty*. Now it is possible to produce other supply functions under quite different assumptions that look very like (3.1). For example, assume that preferences are *implicitly* additively separable so that the life-cycle cost function takes the form

$$c(u, p_1, p_2, \ldots, p_L, w_1, w_2) = \sum_{t=0}^{L} c_t(u, w_{1t}, w_{2t}, p_t) \tag{3.15}$$

where u is lifetime utility. Such preferences are *not* equivalent to the additively separable preferences of (2.1) and have quite different behavioral consequences; see Gorman [**21**, **22**] and Deaton and Muellbauer [**14**, Chapter 5]. Male labor supply in period t is given by

$$h_{1t} = T_{1t} - \frac{\partial c_t(u, w_{1t}, w_{2t}, p_t)}{\partial w_{1t}} = f^*_{1t}(u, w_{1t}, w_{2t}, p_t). \tag{3.16}$$

Under certainty, u is fixed for the whole life cycle so that, apart from the substitution of u for r, (3.16) and (3.1) are identical. Hence, if we follow MaCurdy's suggestion and treat r in (3.1) as an unobservable fixed effect, we have no means of knowing whether we are estimating Frisch demands as in (3.1) or Hicksian demands as in (3.16) even though the interpretation of the results would be different in the two cases. Other cases can also be generated.

Under uncertainty, it is more difficult to think of alternative sensible models that generate the Frisch demands together with the behavior of the utility price. For instance in the example of (3.15), *inter*temporal preferences are Leontief so that, under uncertainty, an individual who consumed too little early in life would throw away much of his or her later wealth. Indeed, it is difficult to think of any simple characterization of intertemporal choice under uncerainty *without* intertemporal additivity. Even so, the fact that there may exist alternative interpretations of our equations does not threaten their validity. If our life-cycle model is correct, our equations are the appropriate ones to estimate and if they cannot describe the data, then the theory is false. If they do describe the data, the model may be true or something else may be true; this is the normal situation.

4. COMPARATIVE STATICS, DYNAMICS, AND THE ROLE OF CHILDREN

Many of the general characteristics of the class of models discussed here are familiar from the work of Ghez and Becker. Here we look only at those results that are useful later. We also give examples of how one might develop profit functions suitable for empirical implementation. We look first at "cohort effects," differences in behavior predicted for households of different ages at a single moment in time, turning secondly to the analysis of individual life cycles.

Individuals born at different dates face different economic environments throughout their lives. In the models here, these cohort effects show up as differences in r, the price of lifetime utility. In general, we think of younger consumers as being better-off; they typically face higher real wages than did their parents at the same age and they inherit or expect to inherit more assets. Given concave utility, being better-off drives up the price of utility. On the average then, we should expect r to increase as we move from older to younger households. These effects are worth noting formally. They all derive from the budget identity for the Frisch demands, i.e., from

$$\text{(4.1)} \qquad \sum_t \hat{w}_{1t} f_{1t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) + \sum_t \hat{w}_{2t} f_{2t}(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) + \sum_t p_t \cdot \phi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = W_0$$

where W_0 is assets at birth, that is, the present discounted value of present and future time endowments and of future assets to be inherited. This identity implies:

(a) Growth in real wages increases r. It is clear, given zero homogeneity, that this lowers the effective price for goods, thus increasing real consumption throughout the life cycle. The net effects of $\hat{w}_{1t}/r$ and $\hat{w}_{2t}/r$ depend on exactly how wages rise and on the value of assets. Hence, as one would expect, the effect of increasing real wages on labor supply is not theoretically predictable. In this between cohorts context, the analysis is the standard one with offsetting income and substitution effects.

(b) Fully anticipated inflation is neutral if assets at birth are indexed. Otherwise there are real balance effects of the traditional kind.

(c) Increases in inherited assets increase r and so decrease both participation and hours and increase real consumption given the normality of consumption and leisure. We should thus expect older workers to have lower lifetime consumption expenditures because of their lower lifetime real wages and asset levels at birth. If there is real asset accumulation over time we should also expect them to work more lifetime hours than their younger counterparts.

(d) Growth in w_2/w_1 is likely to increase female participation and hours relative to male participation and hours. We can thus expect higher participation rates among younger than among older female workers.

Consider now the evolution of family labor supply and consumption over the life cycle. At the general level note the usual life-cycle model disassociation of income and consumption. Adults work hardest when (discounted) wages are highest, typically somewhat before the peak in lifetime wage rates, and not when they have the greatest need for income. The presence of children affects consumption and the time allocation of the parents only in so far as children's time and goods requirements are typically age specific and are not substitutable across periods. Whether or not the births of children are unanticipated, their subsequent development most surely is so that the general income effects of children are captured by the lifetime utility price. Parents work hard to support their children, but in a world without market imperfections, there is no need to do so at the precise moment when the children are the greatest financial burden. It is much better to earn and save when the wage is highest, or to borrow against that period.

A fairly general example illustrates many of these points. Write the period t profit function in the form

$$\pi_t(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t) = \alpha(r, \hat{w}_{1t}, \hat{w}_{2t}, p^*(a_t)) - \beta(r, \hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t, a_t) \tag{4.2}$$

where a_t is a vector of demographic characteristics of the household at time t. For example, the vector might comprise two elements, a_1, the number of small children, and a_2 the number of large (older) children. The $\beta(\ )$ function represents the *costs* imposed on the parents by the presence of children, in terms of both time and goods. The wage rates $\hat{w}_{1t}$ and $\hat{w}_{2t}$ in β emphasize the children's requirements for parental time; one reasonable specification would make husband's and wife's time substitutes in child care so that, for example, all child care would be assigned to the partner with the lowest market wage after correction for "efficiency" in child care. The prices of goods enter costs through needs for child-related commodities; we might specify that older children are commodity intensive and younger children time intensive. Combining these ideas suggests

$$\beta = \{\theta_1(r)a_1 + \theta_2(r)a_2\} \min(\hat{w}_{1t}, \xi\hat{w}_{2t}) + \mu_1(r, p)a_1 + \mu_2(r, p)a_2 \tag{4.3}$$

for efficiency parameter ξ and with $\theta_1 > \theta_2$ and $\mu_1 < \mu_2$. The presence of r allows the costs of children to vary with the lifetime welfare level of the household. Note that this does *not* imply that child costs are not real costs. The household with high life time resources may feel constrained to buy private education for its children; the fees still come out of the parental budget.

The $\alpha(\ )$ function represents the positive side of family life; it is the value of parental pleasures and parental leisure. Here one might expect l_{1t} and l_{2t} to be complementary, at least if the parents enjoy each other's company. If l_{1t} is separable from l_{2t}, the parents enjoy their leisure separately, and the $\alpha(\ )$ functions is additive in $\hat{w}_{1t}$ and $\hat{w}_{2t}$ (note that profits are additive if utility is). The $p^*(a_t)$ function indicates that the presence of children alters the effective price faced by parents for adult goods. We have in mind the Barten [**8**] model in which

$$p_i^*(a_t) = p_{it} m_i(a_t) \tag{4.4}$$

for scaling factors $m_i(a_t)$; see Deaton and Muellbauer [**14**, Chapter 7] and Pollak and Wales [**43**] for further discussion. To give a concrete example, the cost of a trip to the cinema is increased by the cost of a babysitter when small children are present. Hence, the presence of children not only has direct effects through the costs $\alpha(\ )$ but indirect effects acting through "pseudo" relative price changes on adult consumption patterns. In this sense, the model (4.2) is an application of Gorman's [**22**] formulation of child costs to the life-cycle context.

It is of considerable interest to work through the full set of derivatives for (4.2) and thus to trace out the effects of wage and price change and of family development on labor supply and consumption patterns over the life cycle. Here we limit ourselves to two brief examples.

(a) If the wife is allocated all the child-care duties and still participates in the labor market, the complementarity assumption in $\alpha(\quad)$ implies, for anticipated wage changes,

$$(4.5) \qquad \partial h_1/\partial w_2 = \partial h_2/\partial w_1 > 0.$$

The equality is by symmetry. An increase in w_1 results in longer hours for the husband; his absence from the home devalues his wife's leisure and she works more hours in the market. The "independent leisures" assumption would set both derivatives to zero. In most studies of female labor supply based on the static model $\partial h_2/\partial w_1$ is found to be negative. But this is an income effect following from the increase in welfare after the wage increase. In the current context, the wage change is anticipated so that there is no income effect.

(b) The reservation wage of the wife, w_t^*, defined by (2.8) varies positively with the number of small children, i.e., $\partial w^*/\partial a_1 > 0$. Hence, an increase in the number of infants will decrease hours of women who continue to participate in the market and will increase the probability of nonparticipation. More interesting is the behavior of the husband's hours. Firstly, there is a direct effect through the Barten prices on goods consumption and thence on labor supply if goods and male leisure are not separable. Ignore this for the moment on the assumption that there is such separability. The direct time cost of the infant falls on the wife and, provided she continues to participate in the labor market, there is no effect on husband's leisure or market hours. Given (4.2) and (4.3) with $\xi w_{2t} < w_{1t}$, the husband's Frisch labor supply function is independent of a_1 (except through the Barten effects). However, once the wife ceases to participate, the effects of a_1 on w^* enter the husband's labor supply function. Given the "loving couple" complementarity assumption, $\partial h_1/\partial a_1 > 0$, i.e., the additional (anticipated) infant causes the husband to work longer hours. Essentially, when the woman cannot adjust her hours in the market, extra infants cause her to adjust her hours in the home. The extra time spent with the children leaves less for her husband who responds by working longer hours. It is clearly not necessary to invoke credit restrictions and the need to feed the extra mouths to explain the finding of greater male labor supply in response to an increase in family size.

PART TWO: IMPLEMENTATION

5. FUNCTIONAL FORMS

Our selection of useful functional forms is partly guided by the usual criteria: (a) that they be flexible up to the first derivatives of the demands and (b), that they allow simple parametric testing of important hypotheses, particularly symmetry and separability restrictions between the types of labor and individual commodities. However, we also have an additional requirement, that it be possible and convenient to treat the price of utility as an unobservable fixed effect, with or without random parts. Fixed effects can most easily be dealt with by differencing, provided that they appear additively in the demand and supply functions, i.e., we require Frisch demands of the form (again using h_1 as an example)

$$(5.1) \qquad t(h_{1t}) = \gamma_1(r) + \eta_1(\hat{w}_{1t}, \hat{w}_{2t}, \hat{p}_t)$$

where t is some monotone parameter-independent transformation (e.g., a logarithm) and $\gamma_1(\ )$ and $\eta_1(\ )$ are suitable functions. This formulation is also useful for the uncertainty case since ln r differences to give a sum of an observable and a well-defined stochastic term (see (3.14)) so that $\gamma_1(r)$ will do the same, if not exactly then as an approximation.

In principle, there are a number of choices for the $t(\ )$ function. Heckman and MaCurdy [**32**] and MaCurdy [**37**] use logarithms but this has disadvantages. If log hours is the dependent variable, it is hard to analyze participation since the predicted hours can never be zero or negative; effectively such a choice assumes preferences in which hours are essential. Log leisure is better, but leisure is not directly observable. In practice it is measured by subtracting hours worked from available hours (e.g., 168 hours per week) but assigning a value to this is essentially arbitrary and the results obtained are not invariant to the assignment. The same difficulty applies to budget *shares* as dependent variables. In this sort of model, the denominator of the shares is "full income" which, like leisure, requires knowledge of available hours. Hence, the effective choices for the dependent variable are hours or hours multiplied by the wage rate, i.e., earnings. We analyze both.

To ease notation, we temporarily use the vector q to denote all demands and supplies, i.e., commodities and labor supplies. Similarly the vector p is a vector of the two wage rates and the n commodity prices. For *Case 1*, with hours/quantities the dependent variable, we require

$$(5.2) \qquad \frac{\partial q_i}{\partial p_j} = f_{ij}(p)$$

to be independent of r. For *Case 2*, with expenditures the dependent variable, we require that

$$(5.3) \qquad \frac{\partial(p_i q_i)}{\partial p_j} = f^*_{ij}(p)$$

be independent of r.

Taking *Case 1* first, (5.2) requires, for suitable $\xi_i(\ )$ and $\zeta_i(\ )$ that

$$(5.4) \qquad \partial\pi/\partial p_i = \xi_i(p) + \zeta_i(r),$$

hence

$$(5.5) \qquad \pi(r, p) = a(r) + \xi(p) + \sum \zeta_i(r) p_i.$$

Now $\pi_i(r, p)$ is given by (5.4) so $\pi_{i0} = \zeta'_i(r)$ which must be homogeneous of degree -1, i.e., $\pi_{i0} = -\mu_i/r$ for positive constants μ_i. This implies $\zeta_i(r) = -\mu_i \ln r + \eta_i$. Differentiating with respect to r gives $\pi_0 = a'(r) - \mu \cdot p/r$ which is zero degree homogeneous, i.e., $a'(r) = \alpha$, so that the profit function takes the form

$$\pi(r, p) = \alpha r + \xi(p) + \sum (\eta_i - \mu_i \ln r) p_i. \tag{5.6}$$

Rewrite this as

$$\pi(r, p) = \alpha r + d(p) + \sum \mu_k p_k \ln \left(\frac{p_k}{r}\right) \tag{5.7}$$

where $d(p) = \xi(p) + \sum \eta_k p_k - \sum \mu_k p_k \ln p_k$. Provided $d(p)$ is chosen to be linear homogenous, $\pi(r, p)$ is linearly homogenous and is the profit function that we want; it represents the most general set of preferences yielding (5.2), i.e., hours and commodity equations that contain r only as an additive effect. The (Frisch) demands corresponding to (5.7) are

$$q_i = -d_i(p) - \mu_i \ln (p_i / r) - \mu_i. \tag{5.8}$$

Since $\pi_0 = u$, we have period utility $u = \alpha - \mu \cdot p / r$, so that substituting for r in (5.8) we have the Hicksian demands corresponding to (5.8):

$$q_i = -d_i(p) - \mu_i \ln (p_i / \mu \cdot p) - \mu_i \{1 + \ln (\alpha - u)\}. \tag{5.9}$$

Since the final term in brackets on the right-hand side is monotone in u and does not contain p, it is clear that (5.9) is a system of demands corresponding to *quasi-homothetic preferences.* Hence, the treatment of r as additive in the hours and quantities demanded implies intraperiod quasi-homotheticity, i.e., that for a single consumer hours and expenditures are linearly related to within period full income.

A similar analysis applied to *Case 2*, with expenditures/earnings as the dependent variable yields, instead of (5.7) for *Case 1*, a profit function

$$\pi(r, p) = \alpha_0^* r + d^*(p) + r \sum \mu_k^* \ln \left(\frac{p_k}{r}\right) \tag{5.10}$$

with Frisch demands

$$p_i q_i = -p_i d_i^*(p) - \mu_i^* r. \tag{5.11}$$

The Hicksian demands for within period utility u are easily calculated and once again yield quasi-homothetic preferences. Both cases are therefore restricted in this way. Note, however, that, in the context of flexible labor supply, quasi-homotheticity *does not* imply linear Engel curves for goods in terms of either income or total expenditure.

There is little obvious reason to choose one of these forms rather than the other, though the constant marginal propensities to *spend* in (5.11) may be more familiar than the constant marginal propensities to *consume* in (5.9). However, neither formulation is more flexible than the other and arbitrarily, we have chosen to work with *Case 1* and the profit function and Frisch demands given by (5.7) and (5.8). For the linear homogenous $d(p)$ function in (5.7) we choose

$$d(p) = -\sum \eta_k p_k - \sum \sum \theta_{kj} p_k^{1/2} p_j^{1/2} \tag{5.12}$$

for parameters η_k and $\theta_{kj}=\theta_{jk}$. This choice of $d(p)$ is clearly a second-order flexible functional form.

Substituting into the Frisch demands, reverting to the original notation, and using the forms (3.3) to deal with both the certain and uncertain cases, gives the system (with both partners participating)

$$(5.13)\qquad h_{1t}=\alpha_{1t}+\beta_1\ln w_{1t}-\theta_{12}\left(\frac{w_{2t}}{w_{1t}}\right)^{1/2}-\sum_{3}^{n+2}\theta_{1j}\left(\frac{p_{jt}}{w_{1t}}\right)^{1/2}-\beta_1\ln r_t,$$

$$(5.14)\qquad h_{2t}=\alpha_{2t}+\beta_2\ln w_{2t}-\theta_{21}\left(\frac{w_{1t}}{w_{2t}}\right)^{1/2}-\sum_{3}^{n+2}\theta_{2j}\left(\frac{p_{jt}}{w_{2t}}\right)^{1/2}-\beta_2\ln r_t,$$

$$(5.15)\qquad q_{it}=\alpha_{it}+\beta_i\ln p_{it}+\theta_{i1}\left(\frac{w_{1t}}{p_{it}}\right)^{1/2}+\theta_{i2}\left(\frac{w_{2t}}{p_{it}}\right)^{1/2}$$
$$+\sum_{\substack{3\\ j\neq i}}^{n+2}\theta_{ij}\left(\frac{p_{jt}}{p_{it}}\right)^{1/2}-\beta_i\ln r_t\qquad (i=3,\ldots,n+2),$$

where the α's and β's bear obvious relationships to the η's, μ's, and θ's, and where, for convenience, the commodities have been renumbered from 3 to $n+2$, i.e., male and female labor supply are commodities 1 and 2. The t subscripts on the α's reflect variation in variables other than p, w, and r.

These equations are linear in the parameters and in the $\ln r_t$ and so can be straightforwardly used for estimation and testing. In particular, the following hypotheses are of interest:

(a) *Symmetry*: Frisch symmetry requires $\theta_{ij}=\theta_{ji}$ for all $i, j=1,2,\ldots,n+2$.

(b) *Additivity*: Types of leisure and/or goods are additively separable *within* the period if $\theta_{ij}=0$. Of particular interest is whether or not $\theta_{12}=\theta_{21}=0$, i.e., whether husband's or wife's leisure is separable. For other purposes, e.g., for many aspects of optimal tax theory, we wish to test separability between specific goods and leisure. In the current context, this can be tested by testing $\theta_{1i}=0$ and $\theta_{2i}=0$.

(c) *Intertemporal Substitutability*: Unlike MaCurdy's formulation, elasticities are not parametrized. However β_1/h_{1t} is the estimated elasticity of current hours with respect to anticipated wage changes and is one of the magnitudes on which we focus.

These functional forms can be modified to account for children and other socio-demographic characteristics in a number of ways. The simplest is to make the α_i's functions of these variables allowing also for an "idiosyncratic" error term. Better would be to explicitly model the effects of children through the Barten-type effective prices and through their time costs, as in the previous section. Participation of the wife can be modelled by analyzing (5.14) as a fixed-effect Tobit as implemented by Heckman and MaCurdy [**32**]. However, (5.14) does not yield an explicit solution for the wife's reservation wage w^* nor therefore does it yield explicit solutions for the male labor supply and commodity demand functions when the wife does not participate. Note, however, that if $\theta_{12}=0$, the wife's participation status has no effect on her husband's labor supply. In this case, male hours can be analyzed without reference to female participation.

In the rest of this paper, we shall ignore female labor supply and assume it to be additively separable from both goods and male labor supply; we therefore estimate (5.13) together with an aggregate version of (5.15).

Note finally that while it is convenient to work with demand functions with additive fixed-effects such as those discussed, the choice of such forms is not costless. In particular, both profit functions derived here implicitly involve a particular "normalization" of the within period subutility or felicity functions. These essentially determine the allocation of lifetime wealth between periods so that our choice of form results in a complete specification of lifetime preferences.

6. CREATING PANEL DATA

The Family Expenditure Survey is not a panel; individual households are not followed through time. However, the survey is in continuous operation so that it provides a random sample of the population each year (subject to the exclusion of certain groups; see Kemsley, Redpath, and Holmes [**35**]). Currently, we have access to data for the seven tax years (April 5th to April 4th) 1970/1, 1971/2, 1972/3, 1973/4, 1974/5, 1975/6, and 1976/7. Hence although we cannot track individual households, we can track *groups* of households. In particular, if we take age as age of the household head, we can look at the average behavior of, say, 25 year olds in 1970/1, of 26 year olds in 1971/2, ending up with 31 year olds in 1976/7. If we take the first group to be a random sample of all 25 year olds in 1970/1, of 26 year olds in 1971/2, ending up with 31 year olds in 1976/7, then the tracking through the surveys produces a series of random samples from the *same* cohort. Given linear in parameter functional forms such as (5.13)-(5.15), mean cohort behavior reproduces the form of individual behavior and the cohorts can thus effectively be treated as individuals. If the price of lifetime utility is constant for each member of the cohort from one year to the next, then its mean is constant for the cohort as a whole. Hence, the sample mean from the survey will be a consistent estimator of the same quantity from year to year, with a precision determined by the sample design. Similarly, if the (log) utility price follows equation (3.14) for each household, so does its mean. Hence, for all practical purposes, the cohort means can be treated as panel data. Indeed the constant random resampling eliminates the problems caused by attrition in genuine panels and one can evisage very long "panels" created in this way.

Although the empirical analysis proceeds entirely in terms of cohort means, it is important to note how essential are the individual household data. First, the functional forms in the previous section are linear in parameters, not in data, so that to obtain cohort means, it is necessary to obtain the average of log wages or $(w_1/p)^{1/2}$, not the functions of the averages. All such means are straightforwardly obtained from the individual household data. Second, the sample means are subject to sampling errors and the individual data may be used to provide estimates of these errors. Since covariances as well as variances can be obtained.

TABLE I
NUMBER OF HOUSEHOLDS IN EACH COHORT IN EACH YEAR

Cohort no. &	age 1970/1	70/1	71/2	72/3	73/4	74/5	75/6	76/7
				Manual				
1	18-23	—	—	201	244	269	303	313
2	24-28	276	257	269	235	266	278	250
3	29-33	204	247	231	239	259	257	247
4	34-38	258	258	267	248	243	259	237
5	39-43	263	282	264	242	219	265	238
6	44-48	266	310	267	254	271	281	254
7	49-53	230	297	268	248	230	216	205
8	54-58	236	240	248	238	209	191	214
				Nonmanual				
1	18-23	—	—	79	108	130	168	167
2	24-28	105	147	148	156	158	175	163
3	29-33	119	154	141	115	148	143	159
4	34-38	122	136	133	137	160	118	137
5	39-43	123	159	156	132	131	137	116
6	44-48	121	155	143	155	160	134	107
7	49-53	144	116	130	128	104	127	135
8	54-58	90	107	105	113	90	104	78
Totals		2557	2865	3050	2992	3046	3156	3020

we are in a uniquely favorable position to implement errors in variables estimators.

In practice, one year cohorts yield samples that are too small to give accurate estimates of the sample means. Consequently, we use five-year age bands sub-divided as to whether the head-of-household is a manual or nonmanual worker. We also limit ourselves to households with heads aged 18–58 in 1970/1 who are then aged 24–64 in 1976/7 so that all are in the normal working span in all of the surveys. The sample is also selected in other ways. We look only at households containing married couples, one of which is listed as a head-of-household. We also eliminate those men who are *not* employees, who are listed as not in work last week, or who have wives listed as self-employed. The elimination of the unemployed is potentially the most serious problem since we are effectively assuming that the unemployed are a random sample of all participants; we plan to extend the analysis to do better than this in further work, for example by treating the unemployed as voluntary nonparticipants. Note that *including* the unemployed in the regressions would not be correct, even if the choice of unemployment is voluntary, since zero hours is a corner solution and must be handled as such. Our exclusion procedure means that our analysis of business cycle effects is confined to variations in hours of those who remain in work; fortunately it is well-known that such variations move parallel with variations in employment; see Pencavel [**42**]. Table I gives the age bands for each cohort in the first year 1970/1, together with the number of households sampled from each cohort in each of the seven years. Ideally, since we are sampling from the same underlying cohorts through time, we would expect to get the same size samples from each survey. In practice, this does not happen for a number of reasons. The cohorts themselves will change somewhat through death, emigration, and immigration. More importantly our sample selection criteria do not act randomly, particularly with age. This appears most dramatically for the first two cohorts, particularly nonmanual workers, where selection on marriage and employment status excludes a higher proportion of younger workers. Hence the observed sample sizes increase with age, by about 50 per cent for manual workers and about 100 per cent for nonmanual workers. Even this understates the problem since we have excluded the first cohort in the first two years in an attempt to limit unrepresentativeness. There is also a suggestion of declining sample sizes in the oldest cohort as it approaches retirement in the last few years. Presumably this is partly a result of early retirement, though the FES response rate is known to decline monotonically with age; see Kemsley [**34**]. Whether or not these sampling effects bias our results will of course depend on the relationship between the selection criteria on the one hand and labor supplies and commodity demands on the other.

We note finally that in constructing cohort samples there is a trade-off between cohort size and the number of cohort means. If we had taken one year cohorts from Table I, there would have been five times as many "observations" although each would have had about one fifth the number of observed households. Smaller cohort size implies less precise sample means so the essential trade off is between the number of observations and the accuracy of each. If errors-in-variables estimators are used, it is possible to optimize on this trade off and thus to determine an optimal cohort size. This turns out to be a substantial research project in its own right and the results are reported elsewhere; see Deaton [**13**]. For the rest of this paper, we shall treat sample cohort means as if they were population cohort means.

7. EMPIRICAL RESULTS FOR MALE LABOR SUPPLY AND FOR CONSUMPTION

In this paper, we deal with male labor supply and aggregate consumption only. Results on female labor supply and on disaggregated commodity demands will be presented elsewhere, but even the limited task here is complex enough. Subsection 7.1 is concerned with preliminaries of data construction for the empirical models. Subsection 7.2 deals with the relationship between male hours and wages, both under perfect certainty and under uncertainty. Finally, subsection 7.3 is concerned with the joint analysis of consumption and male hours.

7.1. *Preliminaries*

We begin from the version of (5.13) in which male leisure and consumption are additively separable from female leisure and in which there is a single aggregate commodity q. Write the demand and supply equations *under certainty* as

$$(7.1)\qquad h_{it}^{c}=\alpha_{1t}+\beta_1 \ln \hat{w}_{it}^{c}+\theta_1\sqrt{\frac{p_t}{w_{it}^{c}}}-\beta_1 \ln r_i^{c},$$

$$(7.2)\qquad q_{it}^{c}=\alpha_{2t}-\beta_2 \ln \hat{p}_{it}^{c}-\theta_2\sqrt{\frac{w_{it}^{c}}{p_t}}+\beta_2 \ln r_i^{c},$$

where i is the individual household, "born" at time c (the cohort identifier) and observed at time t. The "1" subscript on male hours has been dropped. In theory $\theta_1=\theta_2$, by symmetry, and we shall be interested to test this. Note that r_i^c is the individual's price of lifetime utility discounted to birth and does *not* have a t subscript. The first step is to average over all i belonging to c which removes the i subscript; (7.1) and (7.2) then hold for cohort means. Note that the means involved are the sample averages of the variables in the equations, not of their components; i.e., the right hand side variables in (7.1) are the means of the logarithms of the discounted wage, of the square root of the price-wage ratio, and so on. To the extent that cohort sample means are error-ridden estimates of the cohort population means, there will be biases in estimation. In particular, it should be noted that samples with abnormally high wages will tend to have abnormally high lifetime utility prices and thus low hours if leisure is normal, so that if $\theta_1=0$, treating $\ln r^c$ as time independent will tend to bias downwards the estimates of β_1 if the model is correct.

The second step is to change to a practical method of discounting. As written above, $\hat{w}^c$ and $\hat{p}^c$ are discounted back to the beginning of cohort c's life, i.e., to c, and this date is different for different cohorts. A more convenient procedure is to write $\tilde{w}_t^c=w_t^c\delta(t, T)$ where T is some fixed calendar date (we use January 1974), and w_t^c is the current nominal wage. Clearly $\ln \hat{w}_t^c=\ln \tilde{w}_t^c+\ln \delta(T, c)$, and similarly for prices, so that if we define $\ln \tilde{r}^c=\ln r^c-\ln \delta(T, c)$, (7.1) and (7.2) become

$$(7.3)\qquad h_t^{c}=\alpha_{1t}+\beta_1 \ln \tilde{w}_t^{c}+\theta_1\sqrt{\frac{p_t}{w_t^{c}}}-\beta_1 \ln \tilde{r}^{c},$$

$$(7.4)\qquad q_t^{c}=\alpha_{2t}-\beta_2 \ln \tilde{p}_t-\theta_2\sqrt{\frac{w_t^{c}}{p_t}}+\beta_2 \ln \tilde{r}^{c}.$$

Note that $\tilde{p}_t$ has no cohort superscript since it is now a common price discounted to a common date. The quantity $\tilde{r}^c$ does not vary with t. However r^c *must* vary with c since for later cohorts lifetime prices will on average be higher as will real resources if there is economic growth. One *possibility* is that r^c varies with c as r_t varies with t, i.e., that $r^{c+1}=(1+i_c)r^c$, so that the price of utility increases from cohort to cohort with prices and with the real interest rate. In this case, $\tilde{r}^c$ is independent of c, and the last terms in (7.3) and (7.4) are absorbed into the constant. We shall not impose this restriction however; in the implementation of (7.3) and (7.4) cohort dummies will be included and their significance tested for, while in the intracohort first-differenced forms all time-invariant cohort specific variables are differenced out. Note in particular that fixed date discounting would lead us to expect no coherent pattern in cohort dummies if the model is true.

In the uncertainty case, we use undiscounted Frisch demands from Section 3. Taking the labor supply equation to illustrate, we write

$$h_t^c = \alpha_{1t} + \beta_1 \ln \tilde{w}_t^c + \theta_1 \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \ln r_t^c \tag{7.5}$$

so that, differencing within cohorts and using (3.14), we have

$$\Delta h_t^c = \Delta\alpha_{1t} + \beta_1 \Delta \ln \tilde{w}_t^c + \theta_1 \Delta \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \eta_t \tag{7.6}$$

which is the first difference of (7.3). The corresponding equation for consumption is the first difference of (7.4), together with the term $\beta_2\eta_t$. Note that the innovation η_t will generally be correlated with the right-hand side variables to the extent that these contain unanticipated components. Instrumental variables are therefore required in this case and natural instruments are available in the shape of quantities known in period t or earlier. Our empirical procedure is therefore a straightforward one. Equation (7.3) and (7.4) represent the model under certainty; their first differences, estimated by instrumental variables, are the appropriate equivalents under uncertainty.

Finally, we modify the equations to allow for the possible effects of variations in household size over the life cycle. This is most conveniently done by treating α_{1t} as a variable to write

$$h_t^c = \alpha_1^0 + \beta_1 \ln \tilde{w}_t^c + \theta_1 \sqrt{\frac{p_t}{w_t^c}} - \beta_1 \ln \tilde{r}^c + \gamma_{11} a_{1t}^c + \gamma_{12} a_{2t}^c + u_{1t}^c, \tag{7.7}$$

$$q_t^c = \alpha_2^0 - \beta_2 \ln \tilde{p}_t - \theta_2 \sqrt{\frac{w_t^c}{p_t}} + \beta_2 \ln \tilde{r}^c + \gamma_{21} a_{1t}^c + \gamma_{22} a_{2t}^c + u_{2t}^c, \tag{7.8}$$

where a_{1t}^c is the cohort mean number of young children (less than 5 years of age) and a_{2t}^c of older children (aged 5–15 in 1970 and 1971 and aged 5–17 from 1972 on). In the experiments in this paper we shall treat a_1 and a_2 as if they were exogenous. This can be objected to on the ground that the timing, spacing, and numbers of births is jointly endogenous over the life cycle with labor supply. While acknowledging this in principle, we believe that the feedbacks from labor supply are likely to be less important for male than for female labor supply. Note also that we work with cohort means, where the patterns of young and old children have relatively more to do with human biology and less with economics than would be the case with individual data. It should also be borne in mind that one of the most obvious alternative hypotheses to the simple life-cycle theory of hours is that, in the absence of access to good capital markets, main earners must work to support their offspring and that they therefore work the longest hours when their needs are greatest.

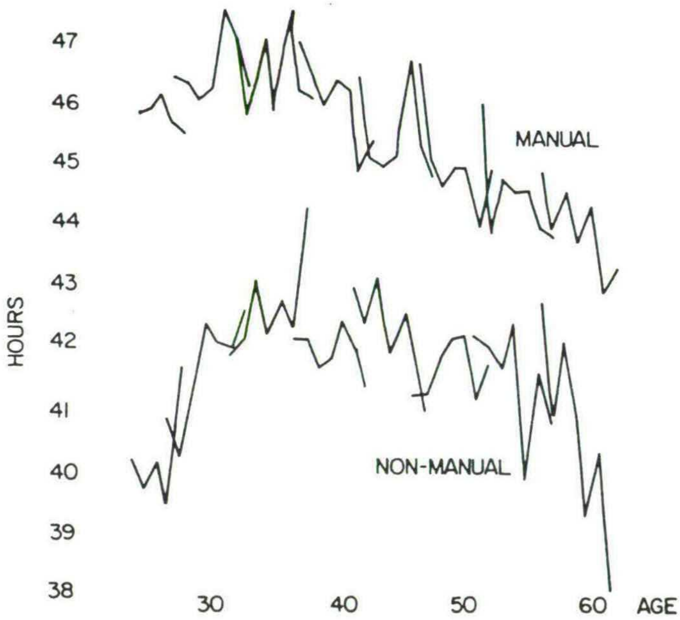

FIGURE 1

The cohort means of hours, discounted wages, prices, price-wage ratios, children, and age of household head are presented in the Appendix, where precise definitions are also given. It should be noted that hours are "normal hours", the definition of which is left to the respondent, while the wage is "normal" income divided by normal hours. (Annual hours are not available in the FES.) The division bias introduced by this construction will tend to impart a downward bias to the hours/wage relationship; this should be offset against the upward bias that results from the inclusion of some overtime in hours and in earnings.

7.2. *Male Labor Supply*

We begin by making the temporary assumption that goods and male labor supply are additive within periods so that $\theta_1 = \theta_2 = 0$. The resulting relationship between h and $\ln w$ is graphed in Figures 1 and 2 which show hours and discounted wages against age for manual and nonmanual workers with each cohort shown separately. As we move from left to right we follow the first cohort as it ages from about 22.5 years in 1972/3 to 26.5 years in 1976/6. The line then breaks to the first observation on the second cohort which in 1970/1 had an average age of close to 26 years. This cohort overlaps with the first cohort for two age observations and we follow them for seven years, breaking off and going back two years in age to pick up the next (third) cohort, and so on. These figures show, in rather noisy form, the traditional life-cycle wage/hours relationships,

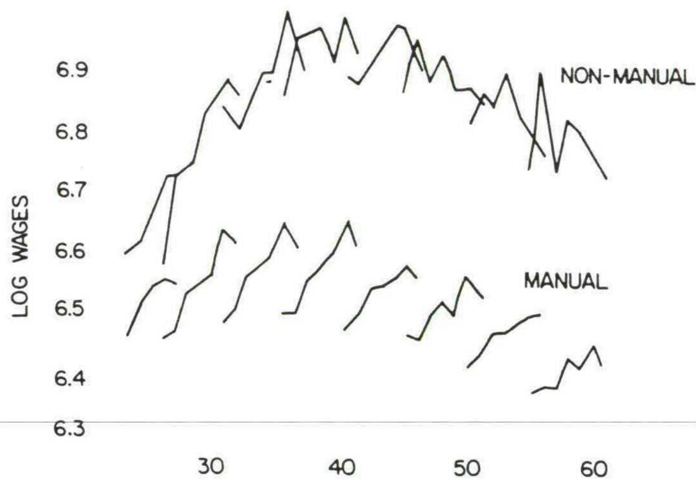

FIGURE 2

and conform to the stylized facts found for the U.S. by Ghez and Becker [**18**] and by Smith [**45**], using cross-sectional data. Manual hours peak early in the life cycle, at age 30-35 and decline fairly steadily thereafter, the total decline being around 3 hours per week. Discounted wages (we use the consol rate) for manual workers also peak early (but somewhat later than do hours) at around 35–40; the total decline is about 14 per cent. For nonmanual workers the pattern is different but there is still a high degree of synchronization between hours and wages over the life cycle. For these workers, hours rise for a longer period reaching a peak around age 35 and declining thereafter; their wages do likewise. But these early life-cycle rises in both hours and wages for nonmanual workers should be treated with particular caution. Our selection criteria exclude such people as students until they enter the labor force. Individuals with high lifetime wage rates will therefore tend to be underrepresented in the earlier years. Even so, the evidence is apparently consistent with the simplest version of life-cycle labor supply. The higher hours by manual workers at all points in the cycle in spite of their lower wages can straightforwardly be ascribed to the domination of income effects between different life cycles, while, as the theory predicts, substitution effects dominate within the cycle.

Table II presents somewhat more formal evidence. Regressions 2.1 and 2.6 confirm the simple correlations over the life cycle between hours and wages for both manual and nonmanual workers. The corresponding (anticipated) intertemporal substitution elasticities, evaluated at the means, are 0.15 and 0.14 respectively, figures close to those calculated by MaCurdy [**37**] for the U.S. However, these results are not very robust. For manual workers, the numbers of young and older children are much more satisfactory predictors of life-cycle labor supply than is the discounted wage; see Figures 3 and 4 below for the behavior of children over the life cycle. The introduction of cohort dummies (the *c* column)

TABLE II

LABOR SUPPLY REGRESSIONS: LEVELS

	Parameter Estimates constant	$\ln \bar{w}$	a_1	a_2	F ratios c	y	R^2	d.w.
			Manual workers					
2.1	−0.08	6.99 (1.86)	—	—	—	—	.213	1.03
2.2	59.18	−2.37 (1.72)	2.21 (0.26)	1.02 (0.18)	—	—	.701	1.96
2.3	88.97	−7.06 (2.98)	0.29 (1.30)	1.34 (0.50)	0.97 (7, 43)	—	.742	1.99
2.4	20.60	3.57 (3.57)	1.71 (0.34)	0.58 (0.29)	—	7.39* (6, 44)	.851	1.74
2.5	80.78	−5.84 (5.22)	0.11 (1.08)	1.35 (0.50)	1.47 (7, 37)	7.50* (6, 37)	.883	1.74
			Nonmanual workers					
2.6	0.98	5.95 (1.33)	—	—	—	—	.278	1.31
2.7	20.61	2.96 (2.10)	0.77 (0.41)	0.67 (0.33)	—	—	.384	1.31
2.8	47.55	−1.07 (2.28)	1.79 (1.17)	2.61 (0.75)	2.72* (7, 43)	—	.574	1.10
2.9	6.47	5.07 (2.30)	0.82 (0.42)	0.42 (0.35)	—	1.21 (6, 44)	.472	1.38
2.10	29.09	1.63 (2.76)	1.10 (1.40)	2.54 (0.80)	2.60* (7, 37)	1.26 (6, 37)	.646	1.27

NOTES: Standard errors are in brackets beneath parameter estimates. Degrees of freedom beneath F ratios: a star indicates significance at a 5 per cent level. Observations are weighted by the square root of cohort size.

has little effect except on the coefficient on a_1; not surprisingly, knowledge of a_1 essentially identifies the cohort and *vice-versa*. However, year dummies are of considerable importance as is to be expected given the clear cyclical effects on both manual hours and wages in Figures 1 and 2. It is clear therefore that the behavior of manual workers' wages and hours over the business cycle is *not*

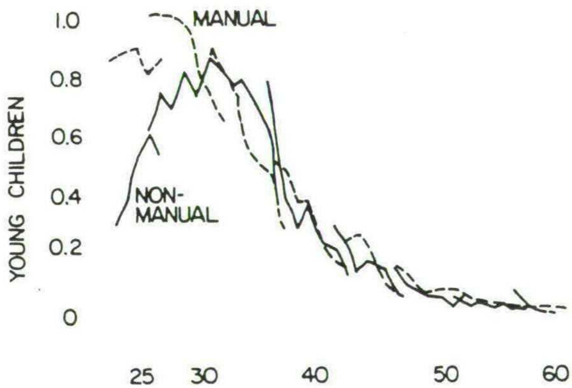

FIGURE 3

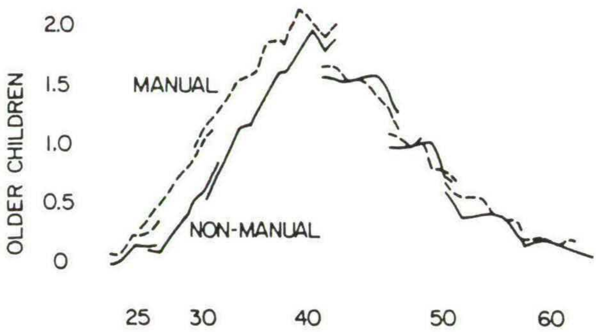

FIGURE 4

explicable by discounted wage variations in a life cycle context, and while the intercohort variation in hours over the life cycle is consistent with the variation in wages, it is much better explained by nonwage variables, such as cohort dummies or, essentially equivalently, by life-cycle variations in household size. The picture for nonmanual workers is similar with some variation in detail. Children and cohorts are somewhat less collinear and business-cycle effects play no role. Regression 2.8 with cohort dummies but no year dummies effectively tells the story; the joint influence of numbers of children and pure cohort dummies leave no significant role for wages in the explanation of hours. For both sets of workers, additional children, particularly older children, exert a consistently positive effect on hours. Such a finding is, of course, explicable from a number of different theoretical viewpoints.

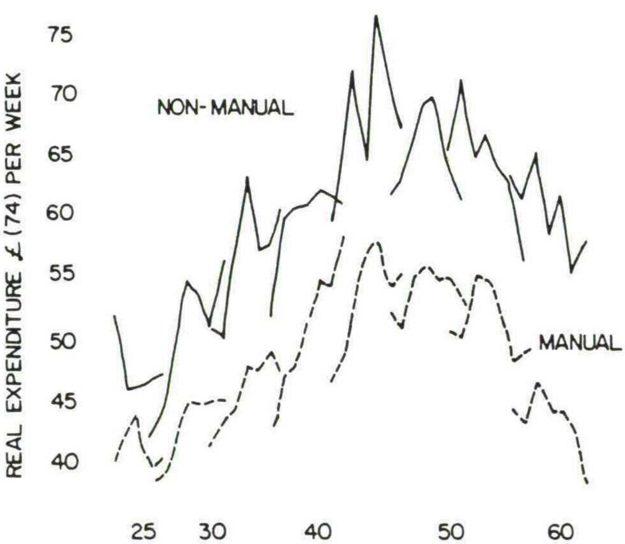

FIGURE 5

TABLE III

POOLED LABOR SUPPLY REGRESSIONS: ALL WORKERS PARAMETER ESTIMATES

	Constant	$\ln \tilde{w}$	a_1	a_2	*NMD*	*F* ratios c	y	R^2	d.w.
3.1	3.81	6.39 (1.11)	—	—	−5.89 (0.41)	—	—	.812	1.51
3.2	33.3	1.67 (1.30)	1.53 (0.23)	0.77 (0.17)	−4.15 (0.47)	—	—	.879	1.61
3.3	41.9	0.265 (1.27)	3.05 (0.63)	2.04 (0.41)	−3.40 (0.48)	2.72* (7, 96)	—	.901	1.70
3.4	10.6	5.17 (1.38)	1.31 (0.21)	0.44 (0.17)	−5.35 (0.49)	—	6.31* (6, 97)	.910	1.66
3.5	16.5	4.19 (1.54)	1.39 (0.77)	1.47 (0.40)	−4.87 (0.58)	2.41* (7, 90)	4.52* (6, 90)	.924	1.67
3.6	−.306	7.29 (1.29)	—	—	−6.19 (0.44)	6.09* (7, 92)	8.13* (6, 92)	.913	1.58

Table III gives results for pooling manual and nonmanual workers allowing for a shift-term in the intercept only, shown in the table as NMD and taking the value 1 for nonmanual workers and 0 otherwise. The covariance analysis does not reject the restriction implied by pooling except, of course, for the highly significant intercept dummy. These results are rather more favorable for the theory although the year dummies are still significant; behavior over the business cycle is not explicable in terms of life-cycle intertemporal substitution. However, in these pooled regressions, and once year dummies have been allowed for, the wage rate has a significant role to play in explaining variations in hours (lines 3.4 and 3.5). Once again, numbers of children have a significantly positive effect on men's hours. Line 3.6 of the table shows the consequence of deleting the child variables; although the restriction is rejected, the only effect is to increase somewhat the wage elasticity as well as the added effect for being a manual worker.

Table IV reports the results of estimation in first differences, both by ordinary least squares and with the wage variable instrumented using variables theoretically uncorrelated with current innovations. These results are once again not particularly supportive of the theory.

Using OLS, the wage is only significant when negative, and only older children and the year dummies retain their previous roles. Indeed, for manual workers alone (not shown), there is a strong negative relationship within cohorts between changes in hours and changes in wages; in the pooled sample, the sign remains but the significance is lost. With instruments, only the year dummies remain significant, although there is still some evidence of a positive influence for older children. The major feature of the pooled first differences is that there is an essentially random scatter between changes in hours and changes in discounted wages. This could arguably be attributed to our sampling procedure for cohorts; first differencing of sample means may generate an adverse signal to noise ratio (a criticism, interestingly, that is often leveled against household *panel* data). But this is not the whole story. Our data as presented in Figures 1 and 2 and in the

TABLE IV

LABOR SUPPLY INTRA-COHORT FIRST DIFFERENCE REGRESSIONS: ALL WORKERS

OLS	Constant	$\Delta \ln \tilde{w}$	Δa_1	Δa_2	*NMD*	y	R^2	d.w.
4.1	−0.15	−4.13	—	—	0.15	—	.051	1.68
	(0.13)	(2.07)			(0.20)			
4.2	−0.18	−4.37	−0.29	1.58	0.13	—	.103	1.80
	(0.14)	(2.06)	(1.33)	(0.73)	(0.20)			
4.3	−0.02	−1.28	−0.24	1.49	0.15	4.05*	.281	1.71
	(0.25)	(2.23)	(1.28)	(0.70)	(0.18)	(5, 82)		
IE								
4.4	−0.15	−4.08	—	—	0.15	—	.032	1.68
	(0.14)	(2.80)			(0.20)			
4.5	−0.18	−4.53	−0.27	1.58	0.13	—	.085	1.80
	(0.15)	(2.81)	(1.35)	(0.74)	(0.20)			
4.6	0.13	3.77	−0.42	1.42	0.18	2.80*	.273	1.75
	(0.27)	(3.80)	(1.33)	(0.73)	(0.19)	(5, 82)		

NOTE: The $\Delta \ln \tilde{w}$ variable was instrumented using age, age^2, and all one-period lagged prices and wages.

Appendix are consistent with those of other researchers, and such figures have typically been cited as evidence in favor of life-cycle theory. However, Table IV tells us that the figures essentially illustrate a simple positive correlation between hours and wages *across* cohorts. As to behavior *within* cohorts, there is little or no evidence in favor of the theory. Indeed the positive relationship between wages and hours over the life cycle stands in contrast to the essentially negative relationship revealed by year-to-year changes.

7.3. *Labor Supply and Commodity Demands*

The previous subsection imposed the restriction that goods and male labor supply are additive within periods; we now relax the assumption. This allows us to see whether allowing for intratemporal substitution affects the previous negative conclusions concerning male labor supply and whether the behavior of consumption itself is in accord with the theory. Once again, we start from the results under certainty reported in Table V. These reveal a number of interesting relationships, though few would strengthen our beliefs in the life-cycle theory. In interpreting these results it should be borne in mind that, apart from minor variations in interview dates, both the consumption price p_t and the discount factor are the same for all cohorts, varying only from year to year. In consequence, regressions containing $\ln p_t$ cannot also contain all the year dummies, and regressions that contain $\ln \tilde{w}_t$, $\sqrt{(p_t/w_t)}$ and year dummies are identified only by functional form, that is essentially not identified.

Looking first at the hours results and comparing with Table III, note that the term $\sqrt{(p/w)}$ is not significantly different from zero in either (5.1) or (5.2) nor is there any evidence of the required positive intertemporal elasticity with respect to wages. The regressions (5.3) and (5.4), which contain the year dummies, tell

TABLE V

HOURS AND CONSUMPTION: ALL WORKERS: LEVELS

				Hours, mean = 43.6						
	Constant	$\ln \tilde{w}$	$\sqrt{p/w}$	a_1	a_2	*NMD*	c	y	R^2	d.w.
5.1	55.5	−0.62	−5.70	1.41	0.69	−4.43	—	—	.883	1.63
		(1.9)	(3.6)	(0.24)	(0.18)	(0.50)				
5.2	55.1	−1.08	−3.47	2.83	1.96	−3.60	2.75*		.903	1.77
		(1.9)	(3.6)	(0.67)	(0.41)	(0.52)	(7, 95)			
5.3	−66.5	13.6	17.4	1.52	0.57	−4.97	—	5.54*	.913	1.68
		(4.9)	(9.8)	(0.24)	(0.18)	(0.53)		(6, 96)		
5.4	−102	17.2	26.0	1.01	1.49	−4.52	2.92*	5.60*	.929	1.69
		(5.5)	(10.5)	(0.78)	(0.39)	(0.58)	(7, 89)	(6, 89)		

				Real Expenditures, mean = 53.3					
	Constant	$\ln \tilde{p}$	$\sqrt{w/p}$	a_1	a_2	*NMD*	c	R^2	d.w.
5.5	117	−10.0	—	—	—	11.6	—	.427	1.16
		(17)				(1.3)			
5.6	212	−23.6	—	−13.3	4.74	11.6	—	.781	1.28
		(11)		(1.3)	(0.66)	(0.82)			
5.7	156	−16.2	—	−6.01	7.68	12.5	5.62*	.845	1.62
		(9.9)		(3.4)	(1.8)	(0.82)	(7, 96)		
5.8	−133	14.9	94.2	—	—	−3.17	—	.580	1.32
		(15)	(15)			(2.7)			
5.9	−56.6	3.67	102	−14.5	0.40	−5.07	—	.867	1.90
		(8.9)	(13)	(0.99)	(0.74)	(2.2)			
5.10	−41.5	2.46	89.2	−12.3	1.61	−2.76	5.03*	.903	1.61
		(8.2)	(12)	(2.8)	(1.7)	(2.1)	(7, 95)		

a different and apparently more attractive story. There is a significant intertemporal substitution elasticity of around 0.4 (holding the current period ratio p/w constant), and there is a significant cross-price effect. By this, in periods when goods are relatively expensive relative to leisure (i.e., the real wage is low), hours (leisure) are relatively high (low); there is significant (specific) *complementarity* between leisure and goods. However, all this is not hard to disbelieve. As before, the presence of the year dummies absolves the economic variables from explaining the year to year variations in hours. Furthermore, the equation is close to being unidentified and the *net* elasticity with respect to wages, taking together the effects of $\ln \tilde{w}$ and $\sqrt{(p/w)}$ is only 0.007, in conformity with the previous results.

The lower part of Table V gives results for real consumption. Lines 5.5 through 5.7 detail the case where goods and hours are assumed separable. This is the least interesting model since prices, unlike wages, do not follow any pronounced life-cycle pattern. Even so price enters with a negative sign that is close to significance when the child variables are included. Either these or the cohort variables have a consistently important influence as is to be expected from the patterns illustrated in Figures 3 to 5. The largest (negative) coefficient (line 5.6) on $\ln p$ suggests an intertemporal elasticity for consumption of around one-half. However, note the persistent negative sign on a_1. This is not consistent with the notion that children carry with them certain age-specific needs. More likely is the alternative explanation, inconsistent with life cycle theory, that current *family income* should play some role in determining expenditures; a_2 is high when male income is at its peak and a_1 is high when female income is low or nonexistent so that their signs are consistent with income being a relevant omitted variable. Lines 5.8 through 5.10 extend the story. The $\sqrt{(w/p)}$ variable is highly significant and has a *positive* sign, so that, according to the consumption side of the picture, leisure and goods are *substitutes*, not complements as is suggested by the hours results. The introduction of $\sqrt{(w/p)}$ also renders the price term positive and insignificant. The effect of older children is also negated as we should expect if, as argued, a_2 is a proxy for male income which, in turn, is not unrelated to (w/p). Note that while year dummies cannot be included in regressions together with $\ln \tilde{p}$, it is possible to compare the performance of $\ln \tilde{p}$ with a complete set of dummies. Perhaps surprisingly, the year dummies are never jointly significant in these regressions and the specialization required to represent all year effects by the single price term cannot be rejected.

Finally in Table VI, the instrumented first-difference regressions are given for both hours and consumption. Although there are minor differences as compared with the levels, the overall pattern is the same. There is no coherent explanation for hours that works both for life and business cycles. Once year dummies are allowed however, some positive wage effects re-emerge even once intercohort variations have been differenced out, while hours again respond positively to the goods price. Once again, the identification of this equation is dubious. On the consumption side, the price effects are barely significant, but once again, the cross-effects operate in an exactly contrary manner to their operation in the hours equation. This lack of symmetry in both levels and differences is more than an

TABLE VI

HOURS AND CONSUMPTION: INTRA-COHORT FIRST-DIFFERENCES BY INSTRUMENTAL VARIABLES

	Constant	$\Delta \ln \tilde{w}$	$\Delta\sqrt{(p/w)}$	*Hours* Δa_1	Δa_2	*NMD*	y	R^2	d.w.
6.1	−0.15	−7.10	−5.76	−0.28	1.52	0.14	—	.096	1.76
		(3.8)	(5.7)	(1.3)	(0.74)	(0.2)			
6.2	−0.02	15.1	24.3	−0.53	1.31	0.12	3.86*	.301	1.73
		(8.0)	(15.2)	(1.3)	(0.71)	(0.2)	(5, 81)		

	Constant	$\Delta \ln \tilde{p}$	$\Delta\sqrt{(w/p)}$	*Real Expenditures* Δa_1	Δa_2	*NMD*		R^2	d.w.
6.3	2.42	−133	—	—	—	−0.18		.086	1.61
		(46)				(0.90)			
6.4	1.62	−101	—	−7.11	6.66	−0.23		.186	1.77
		(41)		(5.2)	(3.0)	(0.77)			
6.5	1.05	−62.5	25.0	−8.32	6.83	−0.16		.227	1.95
		(69)	(38)	(5.0)	(2.7)	(0.7)			

intellectual curiosum or an unimportant deficiency of the life-cycle story. The intertemporally additive models used here have quasi-homothetic preferences within periods so that Gorman [**19**] perfect price aggregation is possible. Hence there exists, for each period, a goods/leisure aggregate that has an intertemporal Frisch elasticity just as do its components, hours and goods. Taking lines 6.2 and 6.5 of Table VI to illustrate the point, the elasticities for hours and for goods are approximately 0.4 and 1.1 respectively. But we cannot talk about *the* intertemporal elasticity because, without symmetry, no aggregator function exists.

In defense of the theory, one final point should be allowed. If uncertainty is taken seriously (as it should be) only Table VI contains fully defensible results. These support our contention that the theory is inadequate, but the standard errors are inevitably large. Even with an initial data set containing nearly 50,000 observations, there is not sufficient information for a really convincing test.

CONCLUSIONS

In this paper we have developed the theory of life-cycle labor supply and commodity demands, making particular use of profit functions to represent intertemporally additive preferences. These profit functions are used as "potential functions" for the marginal utility compensated demand functions of Heckman and MaCurdy, here rechristened Frisch demand functions. We show how the profit and Frisch functions can be used to generate empirically tractable functional forms under both certainty and uncertainty, and we derive the general representation theorems for preferences that allow the price of utility to be treated as an additive fixed effect, as suggested originally by MaCurdy.

Our empirical results are based on seven years of the British Family Expenditure Survey aggregated in such a way as to produce what is effectively panel data on cohort means. Such data bridge the gap between micro and macro and allow a simultaneous analysis of year to year changes (the 'business' cycle) and of variations over life cycles. The British data certainly allow us to tell a coherent story of the life cycle; male wages are closely correlated with male hours and consumption peaks at the peak of real wages, as it should if leisure and goods are substitutes. While this broad sketch cannot be challenged, it does not bear up under closer scrutiny. In particular, the short-run variations in hours are not determined by the same factors as are long-term life-cycle variations; if the life-cycle model is correct for hours, it is only so in the long run and workers are somehow forced off their supply curves in the short run (for example, in an implicit contracts story; see Abowd and Card [**1**]). Secondly, the behavior of hours suggests that hours and goods are *substitutes*; this is inconsistent with the consumption story. Thirdly, the estimated consumption functions strongly suggest at least a partial role for household income; the presence of young children is associated with low not high consumption, and indeed the estimated complementarity of goods and hours is also attributable to alternative explanation in which income plays a role. These results seem to us to be consistent with an emerging consensus based on U.S. data from a wide variety of sources. MaCurdy [**37**], using the data from the Michigan panel study on income dynamics (PSID), comes to different conclusions, but our reading of his results suggests only rather weak evidence in favor of the model. Altonji's [**3**] study is based on the same data, and is a careful attempt to control for the undoubted presence of major errors of measurement. He finds (at best) small substitution elasticities with relatively wide confidence intervals. Ham [**26**], like us, finds significant evidence of labor market constraints. As Ashenfelter [**5**] points out, the "raw" data in the PSID is inconsistent with the model; regressions of changes in hours on changes in wages give persistently negative slopes (see also Ashenfelter and Ham [**4**]), while the ratios of mean change in hours to mean change in wages vary widely from year to year, and are as often negative as positive. Of course, sophisticated econometric methodology can "improve" these results, but the confirmation of the hypothesis is hardly transparent in the data. Aggregate time series tests fare no better; see Altonji [**2**], Hansen and Singleton [**27**], and Mankiw, Rotemberg, and Summers [**39**]; typically, estimated intertemporal elasticities have the "wrong" sign. Finally, and perhaps most convincing, are the *experimental* results from SIME/DIME negative income tax experiments quoted in Ashenfelter [**5**, Table 7]. Households "treated" with artificial guarantees and tax rates reduced their hours relative to controls, and those enrolled in the five-year program did so by more than those enrolled in the three-year program. This is consistent with the existence of life-cycle income effects as predicted by the theory. However, in both three and five year programs, there is no continuing evidence of hours reduction beyond the end of the treatment, contradicting the income effects explanation. It is far from clear what theory would explain this evidence, but it is certainly not the standard life-cycle one. All in all, we believe that these studies, together with the evidence of this paper from Britain, cast a great deal of doubt on the simple life-cycle model that is examined in this paper.

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APPENDIX

THE DATA

The tables below present the cohort averages for each variable for manual and then for nonmanual men.

Definitions

h_1: Normal weekly hours, FES code A220.

w_1: Normal net weekly wage/salary divided by normal weekly hours all multiplied by the discount factor, δ.

δ: The inverse of the product of monthly yields on consols, i.e.,

$$\delta_t = \left\{ \prod_{i=1}^{t-1} (1 + r_i/12) \right\}^{-1}$$

where r_i is the annual yields on consols in month i.

a_1: Number of children aged under 5 in households, codes A040 and A4041.

a_2: Number of children aged 5-17 (to 1972, 5-15 from 1973), code A042.

age: Age of head of household, code A005.

APPENDIX TABLE I

Manual Males

Average weekly hours								Table A1
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	46.4	47.0	46.9	46.5	46.7	46.0	44.9
71-2	—	46.3	45.8	46.4	45.1	45.0	43.9	43.9
72-3	45.8	46.0	47.0	45.9	45.0	44.6	44.8	44.6
73-4	45.9	46.2	45.8	46.4	45.2	45.0	44.6	43.7
74-5	46.0	47.5	47.5	46.3	46.7	45.0	44.8	44.4
75-6	45.6	46.9	46.2	44.9	45.4	44.0	44.0	42.9
76-7	45.4	46.2	46.1	45.4	44.9	44.9	43.8	43.3

Average ln (discounted wages)								Table A2
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	6.46	6.49	6.51	6.48	6.47	6.41	6.37
71-2	—	6.47	6.51	6.51	6.51	6.46	6.43	6.38
72-3	6.47	6.54	6.57	6.56	6.55	6.50	6.47	6.38
73-4	6.51	6.55	6.58	6.58	6.55	6.52	6.47	6.43
74-5	6.55	6.57	6.60	6.61	6.56	6.50	6.48	6.41
75-6	6.56	6.64	6.66	6.66	6.59	6.57	6.50	6.46
76-7	6.55	6.62	6.61	6.61	6.56	6.53	6.50	6.42

Average numbers of small children								Table A3
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.99	.87	.50	.33	.15	.07	.02
71-2	—	1.00	.81	.48	.23	.14	.07	.02
72-3	.85	.99	.74	.37	.27	.07	.04	.01
73-4	.88	.95	.57	.38	.19	.07	.04	.02
74-5	.89	.85	.51	.24	.11	.07	.03	.00
75-6	.79	.74	.45	.17	.08	.07	.03	.01
76-7	.85	.64	.28	.15	.05	.05	.02	.00

Average numbers of older children								Table A4
Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.32	1.07	1.83	1.65	1.08	.65	.28
71-2	—	.49	1.22	1.85	1.61	1.05	.56	.19
72-3	.08	.62	1.46	1.81	1.50	.92	.54	.17
73-4	.10	.79	1.54	2.17	1.52	1.03	.53	.20
74-5	.22	.88	1.56	2.01	1.41	.78	.34	.16
75-6	.25	1.17	1.81	1.90	1.36	.79	.32	.21
76-7	.38	1.28	1.89	2.00	1.11	.61	.29	.14

Year/Cohort	1	2	3	4	5	6	7	8
			Average age				Table A5	
70–1	—	26.0	31.0	36.1	40.9	45.9	50.8	56.1
71–2	—	27.0	32.0	36.9	41.9	47.1	51.7	57.0
72–3	23.3	28.0	33.1	37.9	43.0	48.2	52.6	58.0
73–4	24.2	29.0	34.0	38.8	43.9	49.1	53.7	59.1
74–5	24.9	30.1	35.1	40.0	45.0	50.0	54.9	59.9
75–6	25.8	30.9	36.0	41.0	45.8	51.0	55.9	61.1
76–7	26.6	32.0	37.0	41.9	47.0	51.7	56.6	61.9

Year/Cohort	1	2	3	4	5	6	7	8
			Average Real Expenditure				Table A6	
70–1	—	37.9	40.6	42.1	45.8	51.4	49.9	43.1
71–2	—	38.1	43.0	46.6	47.7	50.2	49.7	42.4
72–3	39.1	40.9	44.0	47.1	51.1	54.8	55.0	45.7
73–4	41.3	44.8	47.8	51.0	56.5	55.7	54.4	43.5
74–5	43.5	44.3	47.0	54.4	57.6	54.2	52.2	43.2
75–6	38.9	44.7	48.4	53.8	53.8	54.6	47.9	41.8
76–7	39.7	44.7	46.9	58.1	54.8	52.3	48.0	37.4

Year/Cohort	1	2	3	4	5	6	7	8
			Average $(\text{Price/Wage})^{1/2}$				Table A7	
70–1	—	1.28	1.26	1.25	1.26	1.27	1.31	1.34
71–2	—	1.26	1.24	1.25	1.24	1.28	1.29	1.33
72–3	1.26	1.21	1.20	1.20	1.20	1.22	1.25	1.31
73–4	1.23	1.20	1.18	1.19	1.20	1.22	1.26	1.28
74–5	1.22	1.21	1.19	1.18	1.21	1.25	1.26	1.31
75–6	1.25	1.21	1.19	1.20	1.24	1.25	1.29	1.32
76–7	1.27	1.22	1.23	1.23	1.25	1.28	1.29	1.35

APPENDIX TABLE II

Nonmanual Males

Year/Cohort	1	2	3	4	5	6	7	8
			Average weekly hours				Table A8	
70–1	—	41.0	41.8	42.2	43.0	41.3	42.3	42.8
71–2	—	40.3	42.1	42.2	42.4	41.3	42.1	41.0
72–3	40.2	41.4	43.1	41.7	43.2	41.9	41.7	42.1
73–4	39.7	42.4	42.2	41.9	41.9	42.3	42.4	40.9
74–5	40.2	42.1	42.8	42.5	42.7	42.3	39.9	39.3
75–6	39.4	42.0	42.3	42.0	41.7	41.1	41.7	40.4
76–7	41.8	42.7	44.3	41.4	41.0	41.8	40.8	38.1

Year/Cohort	1	2	3	4	5	6	7	8
			Average ln discounted wages				Table A9	
70–1	—	6.58	6.84	6.86	6.89	6.87	6.81	6.74
71–2	—	6.73	6.81	6.96	6.88	6.95	6.86	6.90
72–3	6.60	6.75	6.86	6.97	6.91	6.88	6.84	6.72
73–4	6.62	6.83	6.90	6.98	6.94	6.93	6.90	6.83
74–5	6.67	6.86	6.90	6.91	6.98	6.87	6.83	6.80
75–6	6.73	6.89	7.00	6.98	6.97	6.87	6.79	6.76
76–7	6.73	6.86	6.90	6.93	6.90	6.85	6.76	6.73

Table A10

Average numbers of small children

Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.61	.87	.75	.28	.14	.03	.06
71-2	—	.73	.79	.39	.21	.08	.03	.01
72-3	.29	.69	.76	.29	.13	.07	.01	.00
73-4	.37	.81	.77	.35	.17	.05	.02	.02
74-5	.52	.71	.69	.25	.13	.04	.01	.01
75-6	.60	.85	.58	.23	.07	.01	.02	.03
76-7	.53	.57	.32	.13	.06	.04	.00	.00

Table A11

Average numbers of older children

Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	.13	.51	1.38	1.58	.93	.53	.30
71-2	—	.10	.68	1.58	1.51	.95	.37	.07
72-3	.01	.24	.93	1.58	1.54	1.02	.37	.18
73-4	.04	.35	1.16	1.80	1.46	1.03	.38	.19
74-5	.18	.53	1.15	1.98	1.60	.93	.40	.14
75-6	.12	.62	1.41	1.75	1.47	.67	.35	.13
76-7	.13	.88	1.60	1.88	1.23	.63	.28	.06

Table A12

Average age

Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	26.2	30.9	36.2	41.2	46.2	50.8	56.0
71-2	—	26.9	31.9	36.8	42.0	47.1	51.7	56.9
72-3	23.6	28.1	33.0	38.0	43.0	47.8	52.5	57.8
73-4	24.5	29.0	33.9	39.0	44.0	49.0	53.7	58.6
74-5	25.3	30.0	35.0	39.9	45.1	50.1	54.9	59.7
75-6	26.2	30.9	36.3	41.1	45.8	51.1	55.6	60.8
76-7	27.0	32.0	36.9	42.1	47.0	52.1	56.7	61.9

Table A13

Average Real Expenditure

Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	41.4	50.2	51.3	59.0	61.2	64.9	62.9
71-2	—	43.9	49.6	59.6	64.2	62.6	71.1	61.1
72-3	51.7	49.5	57.1	60.4	71.5	66.1	64.5	64.9
73-4	45.2	54.6	63.5	60.8	64.1	69.2	66.4	58.2
74-5	45.4	53.5	56.8	62.0	76.2	69.5	63.9	61.6
75-6	46.1	50.2	57.1	61.5	70.8	63.5	62.4	54.9
76-7	46.4	56.1	60.4	60.8	66.7	61.2	55.3	57.1

Table A14

Average $(\text{Price/Wage})^{1/2}$

Year/Cohort	1	2	3	4	5	6	7	8
70-1	—	1.16	1.06	1.05	1.04	1.05	1.09	1.12
71-2	—	1.11	1.08	1.00	1.04	1.01	1.06	1.04
72-3	1.17	1.09	1.04	0.99	1.02	1.04	1.05	1.13
73-4	1.16	1.05	1.03	0.98	1.00	1.02	1.03	1.06
74-5	1.15	1.05	1.03	1.03	0.99	1.05	1.08	1.09
75-6	1.13	1.07	1.02	1.02	1.04	1.09	1.13	1.15
76-7	1.16	1.09	1.07	1.06	1.07	1.11	1.15	1.17

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CHAPTER 3

EATING, DRINKING, SMOKING AND TESTING THE LIFE-CYCLE HYPOTHESIS

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EATING, DRINKING, SMOKING, AND TESTING THE LIFECYCLE HYPOTHESIS*

MARTIN BROWNING

This paper presents some evidence on expenditure patterns over the lifecycle that has a direct bearing on the question of whether households are significantly credit constrained. Our particular test looks at the consumption of food, alcoholic beverages, and tobacco to see whether the consumption of the latter two "goods" falls as couples have children. The latter usually involves a decrease in household current income and an increase in needs. If households are not credit constrained, they should maintain their consumption of alcoholic beverages and tobacco. We find no significant decrease in the consumption of these goods.

I. INTRODUCTION

The most important implication of the lifecycle hypothesis is that the time path of consumption for an individual agent is independent of the time path of income, subject only to the lifetime budget constraint that the latter imposes. The most widely held caveat to this implication stresses the imperfection of capital markets; many imperfections do lead to a direct link between the time paths of income and consumption (see Hayashi [1985] for a survey). This paper presents some evidence on expenditure patterns over the lifecycle that has a direct bearing on this issue.

Consider the case of two young couples, one with children and one without. Typically the former couple will have lower current income if one partner has to drop out of the labor force to look after the children. The couple with children will also have higher "fixed" costs; children have specific needs for a variety of goods that have to be met. Typically, then, couples with children have lower current supernumerary income than couples without children.

As noted above, one implication of the lifecycle hypothesis is that there is no direct link between current expenditure and current supernumerary income (as long as consumption and leisure are

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separable). Under this hypothesis couples with children will maintain their consumption of such adult-only goods as alcoholic beverages and tobacco; if necessary, they will run down savings or borrow to achieve this. In contrast, alternative hypotheses that stress the imperfection of capital markets predict that the consumption of such goods will fall. In this paper we examine the consumption over time of food, alcoholic beverages, and tobacco by some representative households in the United Kingdom. These data allow us to test whether the consumption of alcoholic beverages and tobacco changes as family composition changes.

In Section II we lay out the theoretical underpinnings of the lifecycle hypothesis as we shall use it. This is then used to derive a demand system for the consumption of food, alcoholic beverages, and tobacco. Together these categories account for about one third of total expenditure; restriction to these three goods is dictated by our data.

Section III introduces demographic variables. There has been a good deal of work on the question of the effect of such variables on consumption (see Blundell and Walker [1984] for some recent results and references). Without exception these studies condition on total within-period expenditure. By contrast, we formulate here an "intertemporal demand system." This does not have expenditure on the right-hand side, so that we cannot compute conventional expenditure elasticities, but it does allow us to derive estimates of genuine "full" price and demographic elasticities. These elasticities take account of intertemporal as well as intratemporal substitution.

We would like to estimate our system on panel data with good consumption data. Unfortunately, these do not exist. We do have, however, a continuous survey of family expenditure in the United Kingdom, and we use this to construct synthetic cohorts that we follow through seven years (1970–1971 to 1976–1977). Section IV outlines the method used (it derives from Browning, Deaton, and Irish [1985]) and takes an informal look at the data so constructed. Although these data are noisy, they have some pronounced regularities.

In Section V we estimate the system of demand functions derived in sections II and III. The theory gives us some testable restrictions which the data do not reject. Our principal finding is that our data do not reject the implications of the lifecycle model which assumes perfect capital markets and rational expectations.

We do, however, have some reservations about our conclusions; these are laid out in Section VI.

II. The Consumption of Individual Goods over the Lifecycle

In this section we investigate the theoretical aspects of the consumption of individual goods over the lifecycle. We shall maintain throughout the hypothesis that preferences can be represented by an intertemporally additive utility function and begin our analysis by assuming perfect foresight and perfect capital markets. Consequently, we have a single lifetime budget constraint; let λ be the Lagrange multiplier associated with it. We then have the first-order conditions for each period t:

$$v_i^t(q^t) = \lambda p_i^t, \qquad i = 1, 2, \ldots, n, \tag{1}$$

where $v^t(\cdot)$ is the period t subutility function, q^t is a vector of quantities of goods in period t, p_i^t is the *discounted* price of good i, and the i subscript on $v^t(\cdot)$ denotes a partial derivative. If we now assume that $v^t(\cdot)$ is strictly concave, we can invert the n equations in (1) to derive "constant marginal utility of money" or Frisch demand functions of the form,

$$q^t = f^t(p^t, \lambda), \tag{2}$$

where p^t is a vector of discounted prices.

The advantage of using Frisch demand functions to explore lifecycle behavior is that we can characterize behavior in period t by reference only to within-period variables and λ, which is a sufficient statistic for all extra-period information. In this, of course, λ is no different from the utility level or expenditure in period t; in fact, with a strictly concave subutility function and given prices, these stand in a one-one relationship to each other. Expenditures or utility levels will, however, typically vary from period to period, whereas the variable λ is constant. Thus, we can often treat the marginal utility of expenditure as a fixed effect in econometric work. This insight stems originally from Heckman [1974] and has been widely exploited since.

Treating the marginal utility of expenditure as a fixed effect will be particularly useful if we take a functional form for preferences that has demands as an additive function of λ. In this case we can "difference λ away." The class of preferences that allow such Frisch demands is characterized in Browning, Deaton, and Irish

[1985]. From that class we choose the parameterization,[1]

(3) $$q_{ih}^t = \alpha_{ih}^t + \beta_i \ln p_i^t + \beta_i \ln \lambda_h + e_{ih}^t, \qquad i = 1, 2, \ldots, n.$$

In this parameterization the h subscript refers to household h. The t superscript on the first term on the right-hand side of (3) will allow us to take account of the demographic composition of the household; we discuss this in the next section. The final term in (3) is a conventional white noise error term, assumed uncorrelated with the other right-hand side variables.

The Frisch demand system given in (3) is rather unusual in that it does not have total expenditure on the right-hand side.[2] Thus, there are no "adding-up" restrictions. There is a homogeneity restriction (the coefficients on $\ln p_i$ and $\ln \lambda$ should be equal), but we cannot test it, since λ is unobservable. Attfield and Browning [1985] show how we can use this homogeneity restriction to estimate a Frisch demand system, despite the presence of the latent variable λ. In this paper we follow a more conventional route.

If we drop the assumption of perfect foresight and assume expected utility maximization with an intertemporally additive $V - N$ utility function, then λ is no longer constant from period to period. Instead the evolution of λ_t, the marginal utility of discounted expenditure, satisfies the following Euler condition:

(4) $$E_t\{\lambda_{t+1}/\lambda_t\} = 1,$$

where $E_t(\cdot)$ is the expectations operator conditional on the informa-

1. An earlier version of this paper included $(n - 1)$ relative price terms $(p_j^t/p_i^t)^{1/2}$ in each equation in (3). These terms allow for complementarity or substitutability between food, alcohol, and tobacco. These variables turned out to be jointly insignificant in the empirical work; for the sake of expositional parsimony, we present only the restricted system (that is, (3)). This restriction is equivalent to assuming that intratemporal preferences are strongly separable (that is, $\nu(\cdot)$ is additive).

2. To see the relationship between Frisch (constant λ) and Hicksian (constant utility u) price effects, consider the Hicksian demand function,

$$q_i = g^i(p, u(p, \lambda)),$$

where the level of utility in any period is given explicitly as a function of prices and the marginal utility of expenditure. Taking the partial with respect to p_i, we have

$$\left.\frac{\partial q_i}{\partial p_i}\right|_\lambda = \left.\frac{\partial q_i}{\partial p_i}\right|_u + \frac{\partial q_i}{\partial u}\frac{\partial u}{\partial p_i}.$$

The first term on the right-hand side of this equation gives the conventional compensated price effect. The left-hand side is the "true" intertemporal effect for an anticipated price change.

tion available at time t. We shall approximate (4) by

$$\ln \lambda_{t+1} = \ln \lambda_t + \epsilon_{t+1}, \tag{5}$$

where $E_t(\epsilon_{t+1}) = 0$ incorporates our (rational expectations) assumption that agents use all current information efficiently (see Hall [1978]).

If we now take first differences through (3) and use (5), we have the differenced-demand Frisch system:

$$\Delta q_{ih}^t = \Delta \alpha_{ih}^t + \beta_i \Delta \ln p_i^t + \Delta e_{ih}^t + \beta_i \epsilon_{ht}, \tag{6}$$

where ϵ_{ht} is uncorrelated with any of the anticipated components of the other right-hand side variables. For the system given in (6) we have only one set of theoretical restrictions: the intertemporal own price effect for each good (that is, β_i) should be nonpositive.

III. Demographic Effects on Demand Over the Lifecycle

The last section outlined how we might model the demand for individual goods over time. Looking at (3) again, we see that if real interest rates are positive (so that discounted prices are falling), then we would expect to observe that the demand for any good rises over time if β_i is negative. For individual households, however, the composition of the household also changes over time. These changes may induce rather different time paths for demands.

Our empirical work will be concerned with only three "goods," namely, food, alcoholic beverages, and tobacco. Formally, we assume that each of these three goods is additively separable from all other private goods, leisure, and public goods. We shall also restrict our attention to households consisting of a married couple for which the only demographic changes are the numbers of young children, older children, and adult children. These are defined as children aged less than five, between five and seventeen, and over seventeen, respectively.

Changes in the composition of a household change demand functions in two ways. First, there is a direct or "needs" effect. Children increase the demand for things like food, toys, and other goods in an obvious way. We assume that non-adult children have no direct "needs" for alcohol or tobacco. A less direct route is that suggested by Barten [1964]. Changes in demographics induce effective changes in relative prices. A classic illustration is that parents with young children must pay more to consume ice cream, since the "price" includes the cost of giving their children ice cream as well. We can now consider informally the possible changes in the demands for the three goods that we have chosen to look at, consequent on changes in our demographic variables.

Looking at food, we see that any increase in any demographic variable increases the needs component, although economies of scale in food preparation may reduce the actual effect of this. An increase in the number of children is also likely to raise the perceived price of food. This will be particularly so for older children who are likely to eat much the same foods as their parents, if on a somewhat reduced scale. This effective increase in the price of food results in a substitution effect away from food. Thus, the net result of the needs and Barten effects is ambiguous. We can see, however, that if the substitution elasticity of food is low, then the needs effect will dominate.

If we assume that non-adult children have no "needs" for alcohol or tobacco, then changes in demographic structure can affect the consumption of these goods only via Barten effects. Alcohol can be consumed inside the home or outside (in restaurants and pubs). If the latter is chosen, then the price of "going out" is increased by the price of a babysitter. We note, however, that baby-sitting services are a public good as far as the parents are concerned; that is, a sitter can look after three children as well as one. This suggests that it is the presence of a child that is important and not the numbers. These considerations suggest that alcohol consumption may fall on the arrival of the first child but not thereafter. There do not seem to be any demographic effects for tobacco.

The predictions of the lifecycle hypothesis with perfect capital markets on the effect of more children on drinking and smoking stand in sharp contrast to those of more informal alternative hypotheses. Consider a young couple with a new-born baby. We could plausibly suggest that current income has fallen (as one partner drops out of the labor force) and desired current expenditure has risen due to the specific needs of the baby. The lifecycle hypothesis with the supplementary assumption of perfect capital markets predicts that the household will realize its plans by borrowing against future earnings or by running down savings. Hence the household can maintain its consumption of, among other things, alcohol and tobacco. However, if the household cannot borrow to finance current consumption, then current expenditure will be constrained by current income. Under these circumstances we might expect to see a fall in the consumption of goods like alcohol and tobacco which have no specific "child-needs" component. This dichotomy between the predictions provides us with an opportunity to test the lifecycle hypothesis.

There are, however, two caveats to this. First, people may change their preferences during the pre-natal period or as children are growing up. In particular, the consumption of tobacco and alcohol may fall either from health considerations or from desire to set a good example. In this case households that were not credit constrained would behave as though they were credit constrained, at least as far as the consumption of alcohol and tobacco is concerned. Taking the perfect capital markets case as our null hypothesis increases the power of our test. The second reservation we must admit is that the timing of births is (to a certain extent) a choice variable. Thus, households that are credit constrained may postpone having children until any credit constraint is ineffective. This reduces the effective significance level of our test.

The discussion in this section does not give many predictions. Indeed, about all we can say is that the consumption of tobacco (certainly) and the consumption of alcoholic beverages (probably) will be independent of the presence of non-adult children. It is these "predictions" of the pure lifecycle model that we shall be interested in testing. To do this, we shall let $\Delta\alpha_{ih}^t$ in (6) be given by

$$\Delta\alpha_h^t = \mu_{i0} + \sum_{k-1}^{4} \mu_{ik}\, \Delta a_{kh}^t, \tag{7}$$

where

a_{1h}^t = number of children aged less than five,
a_{2h}^t = number of children aged between five and seventeen,
a_{3h}^t = number of adult males (excluding the husband),
and a_{4h}^t = number of adult females (excluding the wife)
in household h at time t. In these terms our predictions are

$$\mu_{i1} = \mu_{i2} = 0 \text{ for } i = \text{alcoholic beverages and tobacco.}$$

Equations (6) and (7) together lead to a model in which all changes in demands are attributable solely to (discounted) price changes and demographic changes. We do not allow any independent age effects beyond the trend in (3) implicit in the constant in (7), nor do we employ time dummies. The reason for the latter exclusion is that prices within each year are the same across all households so that a system with time dummies would not be identified. Age effects were not included, since our principal interest is the effect of demographic changes on our three categories and age and demographic composition are highly collinear.

IV. An Informal Look at Some U.K. Data

The last two sections presented a framework for thinking about the lifecycle consumption of specific commodities and also indicated that the theory is rather deficient in specific predictions, even when we make strong assumptions. In this and the next section we examine some U.K. data to see whether we cannot answer more specifically such questions as what happens to the consumption of alcoholic beverages over the "lifetime" of a household.

The data we use are taken from the U.K. Family Expenditure Surveys (FES) for the tax years 1970/1971 to 1976/1977. From these surveys we first select households that are headed by a married couple who currently live together. Then we select households where the head of household (always the male in this survey) is working outside the home. This latter is to allow us to determine whether the head of the household is a manual or nonmanual worker; previous work suggested that there are significant unmeasured permanent differences between these groups. In all that follows, we refer to households headed by a manual (nonmanual) worker as a manual (respectively, nonmanual) household. We treat occupation as an exogenous variable.

We construct a cohort by taking means of all of the households with head aged t to $(t + s)$ sampled in 1970/1971, aged $(t + 1)$ to $(t + s + 1)$ in 1971/1972 and so on. For the first manual and nonmanual cohort, we took a six-year band (i.e., $s = 5$); for each of the others, we took five-year bands. Thus, each cohort has a number (cohort 1 is the youngest) and an occupation. We note in passing that if we had taken age bands of 39 years width (that is, one cohort per year), then we would have had conventional (if rather short) aggregate time series.

We have data for seven years on eight age bands and two occupational groups. We shall treat the sample mean of any variable in any year for each of these sixteen cohorts as though it is an observation on a particular household. Thus, we can follow sixteen "synthetic households" through seven years. The graphs of demographic variables against age (not presented here) show an unsurprising pattern: children are born, grow older, and finally begin to leave home. About the only things we have to remark on our data on demographic variables is that nonmanual households appear to start their families later and that there are more adult male children than adult female children in older households.

Figures I to III give the lifetime consumption patterns of food, alcohol and tobacco, respectively. Each figure also has an inset displaying the path of discounted prices over the period 1970–1971 to 1976–1977. The interest rate used to discount prices is the yield on 90-day Treasury bonds; the prices are the implicit price deflators for the three goods. As can be seen, each of the three figures consists of sixteen connected graphs. Each of these connected graphs tracks the mean of the consumption of the particular good for one synthetic cohort over seven years.

Our theory predicts that, ceteris paribus, the pattern of any of the sixteen connected graphs should mirror the pattern of discounted prices given in the inset in the sense that if, as for food, the discounted price is rising, then we would expect all the connected graphs to be downward sloping. Lifecycle patterns may, however, mask this. Suppose, for the sake of illustration, that we had only three cohorts and that our theory is applicable. For a good with a rising discounted price, we would have the pattern shown in Figure

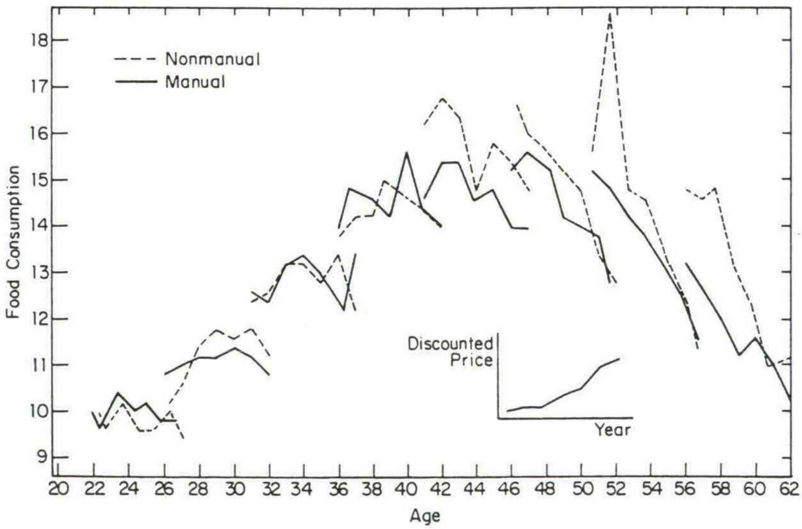

FIGURE I
Food Consumption Over the Lifecycle

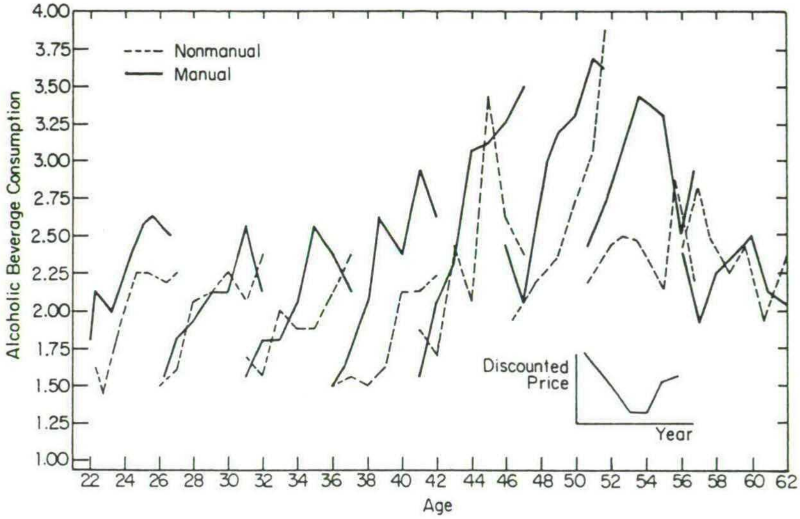

FIGURE II
Alcoholic Beverage Consumption Over the Lifecycle

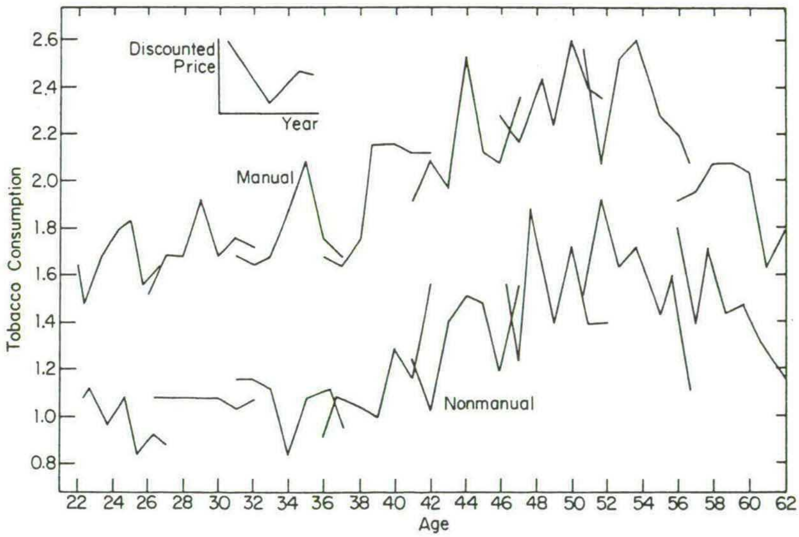

FIGURE III
Tobacco Consumption Over the Lifecycle

IVa. However, if there is also a lifecycle pattern as shown in Figure IVb, then the net effect is as shown in IVc.

In Figure I, the path of the consumption of food, two things are immediately observable. First, the patterns for manual and nonmanual households are very similar. Since manual cohorts are poorer in lifetime terms than nonmanual cohorts, this suggests that the elasticity of food consumption with respect to lifetime wealth is small. Our estimates based on (6) and (7) "difference away" any such permanent intercohort differences. However, undifferenced estimates of our system (not presented here) indicate that nonmanual cohorts consume about 13 percent more than comparable manual cohorts. If lifetime wealth for nonmanual cohorts is about 50 percent higher (so indicated by the earnings profiles in

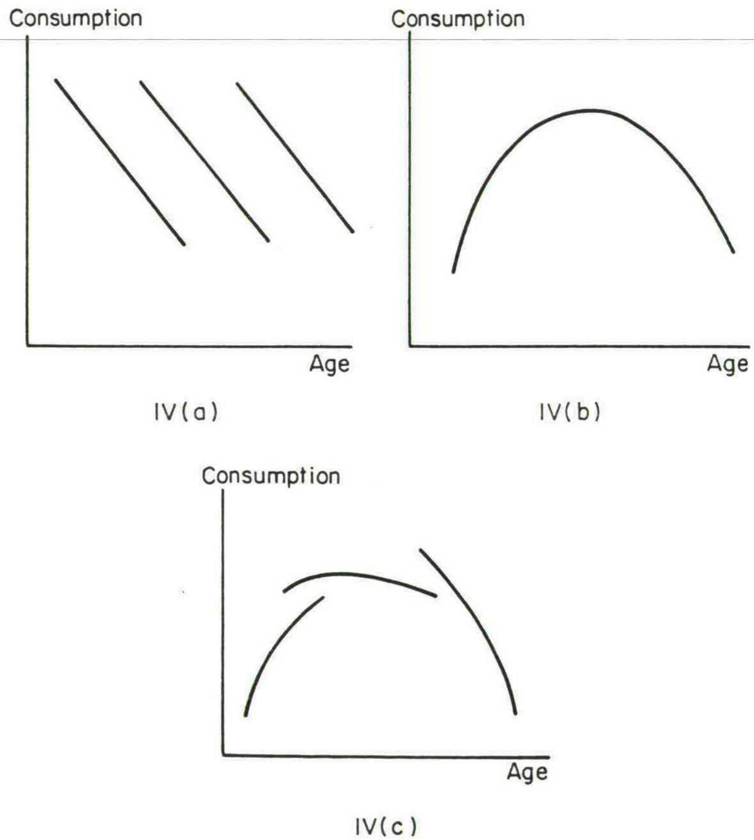

FIGURE IV

Browning, Deaton, and Irish [1985]), this gives a lifetime wealth elasticity of about 0.25. This estimate is not at variance with earlier estimates; for example, Hall and Mishkin [1982] find a "marginal propensity to consume lifetime income on food" of 0.11.

Second, there is a pronounced plateau at about ages 42 to 54 with a steady rise of about 60 percent from age 22 and a steeper fall to age 62. The obvious explanation for this intercohort pattern is that the positive needs of children for food outweigh the negative Barten effects. Notice, as well, that correcting for any year-to-year change due to the steadily rising discounted price would rotate each connected segment counterclockwise which is consistent with the lifecycle pattern. Thus, this figure does not reveal any conflict between the intercohort and the intracohort patterns.

Looking at Figure II, for alcohol, we see first that there is some indication that manual households consume more alcohol than nonmanual households. This suggests either that alcohol is an inferior good in lifetime terms or that preferences are different as between the two groups. The second thing we note about Figure II is that consumption rises from age 42 to about age 53. This is consistent with a number of hypotheses. One would be that alcohol is indeed an inferior good in lifetime terms and older cohorts are poorer. A second explanation is the Barten-type effect given in the previous section; by about age 40 people no longer need to hire baby-sitters. Another explanation would be that younger cohorts are credit constrained and cannot realize their desired consumption of alcohol at the younger ages. None of these, however, can explain the fall later in life. The final explanation is that the older cohorts have adult children, and it is these who are increasing consumption. Note, also, that the discounted price pattern would lead us to predict an inverted-**U** shape for each connected segment.

The third observation we make on Figure II is that it looks like younger cohorts drink more than older cohorts in the sense that each connected graph ends at a higher level than the next connected graph. As an example, it can be seen that a manual household aged 22 at the beginning of the sample consumes about 60 percent more alcoholic beverage at age 26 than does a manual household aged 26 at the beginning of the sample at the same age (compare the end of the first manual connected graph with the beginning of the second manual connected graph). Undifferenced estimates of our system reveal that these differences are significant even when we allow for the increase in consumption over time attributable to the fall over time of the discounted price of alcoholic beverages.

Figure III gives the lifecycle pattern of the consumption of tobacco. We see immediately that manual households consume much more tobacco than nonmanual households. For both groups consumption rises steadily from age 22 through 50 and fall thereafter. Without invoking strong cross substitution or complementarity effects, it is difficult to reconcile the rise earlier in the lifecycle with our theory. The lifecycle pattern within each occupation is, however, partially consistent with strong habit formation. With strong habit formation the "user cost" of establishing a habit falls with expected remaining life span. In such a model more and more agents in a particular cohort would, ceteris paribus, initiate the habit as time went on.

Before turning to a more formal analysis of our data, we must issue a critical warning. It is well-known that the FES underreports the consumption of alcohol and tobacco significantly (see Kemsley, Redpath, and Holmes [1980]). In all of our work we shall ignore this systematic error in our dependent variable. If the errors in reporting are uncorrelated with any of our independent variables, then they will not cause any bias, although they will affect the standard errors of our parameter estimates. However, if the underreporting is correlated with, for example, the number of children, then our estimates will be inconsistent. In this respect, we note that the underreporting seems to be mainly due to the sample design; as it happens, groups known to contain a high proportion of heavy drinkers and smokers are not surveyed.

V. The Estimates of Our System

In this section we present our estimates of (6) and (7); that is,

$$\Delta q_{ih}^{t} = \mu_{i0} + \sum_{k=1}^{4} \mu_{ik}\, \Delta a_{kh}^{t} + \beta_i\, \Delta \ln p_i^t + u_{ih}^t \quad i = 1, 2, 3, \tag{8}$$

where u_{ih}^t is the composite error term from (6). In our estimation procedure we shall assume that

$$E(u_{ih}^t) = 0$$

and

$$\begin{aligned} E(u_{ih}^t u_{jh'}^{t'}) &= 0 \qquad \text{for } t \neq t' \text{ or } h \neq h' \\ &= \sigma_{ij} \qquad \text{otherwise.} \end{aligned}$$

Thus, we rule out any correlation over time or between households.

This latter is likely to be a poor assumption if the variance of $\beta_i\epsilon_h$ is large relative to the variance of e_{ih} and some of the shocks implicit in ϵ_{ht} are common across households. We have found it impossible, however, to allow for common shocks. To see the problem, note that we can break ϵ_{ht} into a common-across-cohorts component and a cohort-specific component. The former component effectively adds a time dummy to each equation; as we have already seen, this leaves the system underidentified if we have a common price in each equation. Thus, our assumptions on the covariance structure of u_{ih}^t are to be seen as identifying assumptions. Note, however, that we do not assume that the errors are independent across goods.

We assume that the composite error term in (8) is uncorrelated with any of the right-hand side variables. This is largely for want of any reasonable instruments. This requires that any shocks in these variables do not lead to revisions in the marginal utility of wealth (although there may be revisions due to income shocks or other surprises). For children already born there are no significant demographic surprises so the assumption is innocuous for Δa_{2h} to Δa_{4h}. Of more concern is the assumption that there are no surprises in Δa_{1h} or the own real rates. For the latter we might assume that any surprises are transitory and do not have much effect on the perceived marginal utility of wealth. For the number of young children we cannot, of course, assume that any surprise is transitory. Effectively, then, we must here assume that desired family composition is predetermined and that family planning is 100 percent effective.

Table I presents the estimates for (8). Table II gives the elasticities and increases in consumption implied by the parameter estimates given in Table I.

The own-price elasticities are all significantly negative. Looking at the other elements of the food equation, we see that older children and adults significantly increase the consumption of food, but young children have no significant effect. Although there is some indication that adult females increase consumption by more than adult males, the estimates for the two are not significantly different. These results suggest that an extra adult male and adult female double food consumption, as compared with a household with just two adults. Two extra older children, on the other hand, only increase consumption by about 50 percent.

Turning now to the other two equations, we recall that predictions about the effect of young children on the consumption of alcohol and tobacco differ sharply as between the lifecycle hypothe-

TABLE I
PARAMETER ESTIMATES FOR EQUATION (10)

i	Food	Alcohol	Tobacco
μ_{i0}	0.08	0.05	−0.01
	(0.11)	(0.05)	(0.02)
μ_{i1}	−0.5	−0.8	0.08
	(0.8)	(0.5)	(0.3)
μ_{i2}	1.97	0.18	0.24
	(0.47)	(0.3)	(0.16)
μ_{i3}	3.66	4.35	1.46
	(1.64)	(1.0)	(0.57)
μ_{i4}	4.65	−0.3	−0.51
	(1.4)	(0.9)	(0.48)
β_i	−8.5	−3.0	−0.86
	(2.3)	(0.86)	(0.29)
R^2	0.4	0.24	0.14

Figures in brackets are standard errors.

sis and our (admittedly rather informally framed) alternative. Our results indicate that children do not significantly reduce the consumption of alcohol or tobacco. This is, of course, what the lifecycle hypothesis with perfect capital markets predicts, but it is at variance with the alternative hypothesis that links current expenditure more closely to current income.

It is also interesting to note that we cannot reject the negative influence of young children on alcohol consumption at a 12 percent significance level, whereas the effects of older children on alcohol consumption and younger children on tobacco consumption are decisively rejected. These results provide some support for the particular variant of the lifecycle hypothesis outlined in Section III. The implied Barten scaling factor for the effect of young children

TABLE II
ELASTICITY ESTIMATES

Good	Intertemporal own price	Percentage increase for one extra: Young child	Older child	Adult male	Adult female
Food	−0.65	−6	23	43	54
Alcohol	−1.28	−29	6.5	158	−11
Tobacco	−0.5	4	12	75	−26

The base level for these calculations is a cohort 4, manual household with no children.

on the price of alcohol is given by the percentage decrease in alcohol consumption divided by the own price elasticity of alcohol. From our estimates this is about 0.26; that is, young children increase the price of drinking by about 26 percent.

Turning to adult consumption of alcohol and tobacco, we note that our estimates may not be well determined due to the collinearity between adult males and adult females. Our parameter estimates indicate that an extra adult male drinks 50 percent more than *both* parents and smokes 75 percent as much, while an extra adult female does not increase the consumption of either good. The parameter estimates for males and females are, however, negatively correlated, so that we cannot reject (more plausible) higher values for adult females and lower values for adult males.

VI. Conclusions

In this paper we have examined the consumption of food, alcoholic beverages, and tobacco over the lifecycle. A demand system that explicitly allowed for uncertainty and changes in demographic structure was estimated.

We feel that there are four principal shortcomings in this paper that need to be addressed in the future. First, it would be desirable to add more structure to the error term in (8). Second, the exclusion of other private goods and leisure has no strict justification beyond expediency. If our three goods and these other goods are not want independent, then we shall have conventional omitted variable bias; this will only be zero if the omitted relative prices terms are orthogonal to our explanatory variables. Third, the additive over time structure for preferences may be particularly inappropriate for alcohol and tobacco, since these are generally considered habit-forming. Finally, our cohorts have not been chosen optimally; it might well be better to have wider age bands and fewer "observations."

Given these reservations, we do conclude that young children do not reduce the consumption of tobacco significantly and reduced the consumption of alcohol only a little. This result is one of the few predictions of our theoretical lifecycle model and stands in contrast to alternative predictions. Thus, these results give some support for the lifecycle hypothesis *in this context*. However, without a coherent alternative to the lifecycle hypothesis, it is impossible to formalize this test.

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CHAPTER 4

A NON-PARAMETRIC TEST OF THE LIFE-CYCLE RATIONAL EXPECTATIONS HYPOTHESIS

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A NONPARAMETRIC TEST OF THE LIFE-CYCLE RATIONAL EXPECTATIONS HYPOTHESIS*

BY MARTIN BROWNING[1]

In a world of perfect certainty and perfect capital markets agents allocate expenditure in such a way that the marginal utility of discounted expenditure is the same in each period. In this paper we present a test of whether any particular series of discounted prices and quantities can be exactly reconciled with a utility function and marginal utility of discounted expenditure that do not change from period to period.

We find that UK, US and Canadian postwar aggregate data all reject our condition, although it is not rejected for long sub-periods. We show that our results for the UK suggest particular modifications to the strong form of the rational expectations hypothesis that our condition tests. In fact these data are exactly consistent with a number of alternative hypotheses. From this we argue that time series data are ill suited to parametric testing of any of the competing hypotheses on the inter-temporal allocation of expenditure.

1. INTRODUCTION

The pure theory of the inter-temporal allocation of goods and time with perfect capital markets and perfect foresight is identical to the static demand case. There is a single (lifetime) budget constraint and the allocation of expenditure to each period is made so that the marginal utility of discounted expenditure is the same in each period. Bewley (1977) and Hall (1978) have identified the life-cycle/rational expectations hypothesis under uncertainty (REH) with the proposition that expected utility maximising consumers will allocate expenditures and time in such a way as to try and keep the marginal utility of discounted money constant.

Given a set of time series on prices, purchases, wages and labor supply we can ask whether these realised plans are consistent with the life-cycle hypothesis. In this paper we propose a test that requires no ad hoc specification of functional forms or error processes but rather gives an answer yes or no based on finite mathematical techniques. The specific hypothesis we test is a strong form of the REH viz that the marginal utility of discounted expenditure is constant over the period of observation. For long time series this seems to be an absurdly strong hypothesis since it is effectively equivalent to assuming perfect capital markets and perfect foresight with regard to all important variables. If, however, we can accept this strong hypothesis for a particular data set then any test of the weaker forms of the life-cycle/rational expectations hypothesis on this data set must be interpreted

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as testing the appropriateness of the functional forms used rather than the hypothesis itself. The particular weaker tests we are thinking of are those that follow Hall (1978), see Hall (1987) for a recent survey. As we shall see, US, UK and Canadian data reject the strong form of the rational expectations hypothesis but rather less often than we might expect.

The (very familiar) basic theory is presented in Section 2. Section 3 develops a test of the strong form of the REH that can be applied to data on (discounted) prices, quantities, (discounted) wages and hours of work. This test is a simple pass/fail one that does not require a particular parameterisation of the underlying preferences. For this reason we follow Varian (1982) in referring to it as a nonparametric test. It has little to do with traditional nonparametric statistics.

Section 4 applies the test to some postwar aggregate data sets for Canada, the US and the UK. We find that none of these satisfy the criterion developed in the previous section for the whole data period but that they do indicate some very strong theory-coherent regularities in the data. In particular we cannot reject the strong hypothesis for long sub-periods, particularly when we allow for shocks in the 1970's. We also find that a test of the SREH on the UK data for adjacent years is rejected for only one pair of years.

Section 5 discusses the results obtained in Section 4 for the UK data. Although these data reject the strong form of the rational expectations hypothesis over the period 1952 to 1985, the rejection is informative. In particular we show that the data can be exactly reconciled with weaker forms of the theory that allow for uncertainty or imperfect capital markets or nonadditive preferences over time. Furthermore we show that the data is also exactly consistent with a Keynesian model that has a conventional Marshallian assumption for intra-period allocation.

Our principal conclusion is that aggregate time series data sets are ill-suited to testing between alternative hypotheses on the inter-temporal allocation of expenditure (and time).

2. THE STRONG RATIONAL EXPECTATIONS HYPOTHESIS

Suppose agents have perfect foresight, face perfect capital markets and have preferences over T periods that can be represented by the intertemporally additive utility function

$$\sum_t v(q^t) \tag{2.1}$$

where q^t is a vector of n goods purchased and consumed in period t and $v(\cdot)$ is concave and strictly increasing. Given a particular allocation of total expenditure to each period we have the first order conditions for optimal intra-temporal allocation:

$$v_i(q^t) = \lambda_t p_i^t, \qquad i = 1, 2, \ldots, n \tag{2.2}$$

where $v_i(\cdot)$ is the partial derivative of $v(\cdot)$ with respect to q_i^t, λ_t is the marginal utility of discounted expenditure for period t and p_i^t is the discounted price of good i in period t.

Inverting the n conditions in (2.2) gives demand curves that are functions of discounted prices and the marginal utility of discounted expenditure. We refer to these as "Frisch demand functions" to distinguish them from Marshallian (prices and parametric expenditure) and Hicksian (prices and parametric utility level) demand functions.

If expenditure in each period is determined arbitrarily then λ_t will vary from period to period. Bewley (1977) and Hall (1978), however, identify the life-cycle hypothesis with the proposition that consumers seek to keep λ_t (which is given in discounted terms) constant from period to period. In our context this leads to the following definition.

DEFINITION. *The time series of discounted prices and quantities* $\{(p^t, q^t)\}$ *satisfy the* Strong Rational Expectations Hypothesis *(SREH) if there is some concave, strictly increasing (utility) function* $v(\cdot)$ *and a positive* λ *such that*

$$v_i(q^t) = \lambda p_i^t, \qquad \text{for all } i, t. \tag{2.3}$$

Clearly the SREH is equivalent to optimal intra-temporal allocation (condition (2.2) for all t) and constancy of λ_t. An alternative way of stating the SREH is that in any period t the agent has no regrets about the choices made in periods 1 to $(t - 1)$. If it transpires that $\lambda_r < \lambda_s$ for some r and s then the agent would, ex post, have preferred to transfer expenditure from period r to period s.

If we are dealing with time-series data over long periods then the requirement that λ_t be constant over time appears very strong. For short-run data, however, it may well be more reasonable to assume that λ_t is constant. Indeed, Bewley (1980) explicitly calls Frisch demand functions with a constant λ "short-run demand functions". This constancy is the reason for the original use of Frisch demand function in Heckman (1974) as well as in, for example, the work of MaCurdy (1981) and Browning, Deaton and Irish (1985).

A weaker version of the SREH is to require it to hold only for adjacent years. This is the hypothesis that agents successfully forecast one year ahead. We shall refer to this as SREH2. Clearly, SREH implies SREH2. The converse is not, however, the case; it may be that the set of utility functions that "rationalise" the data for $(t - 1)$ and t and the set of utility functions that rationalize the data for t and $(t + 1)$ have no common elements.

If we allow for uncertainty then agents maximising expected utility and using all currently available information set:

$$E_{t-1}(\lambda_t) = \lambda_{t-1} \tag{2.4}$$

where $E_{t-1}(\cdot)$ is the expectations operator conditional on the information set at time $(t - 1)$ (see Hall 1978, or Browning, Deaton and Irish 1985). Appropriate tests of this condition require statistical tests based on particular functional forms. The SREH however, is amenable to nonparametric testing. In the next section we derive such a test. Any discussion of the "meaning" of the results of this test are postponed until Section 5, following the implementation of the test for a varied assortment of aggregate data sets.

3. A NONPARAMETRIC TEST OF THE STRONG RATIONAL EXPECTATIONS HYPOTHESIS

In this section we derive necessary and sufficient conditions that a time series of discounted prices and quantities must satisfy to be consistent with the SREH. To derive the necessary conditions, suppose that $\{(p^t, p^t)\}$ do satisfy the SREH for some $v(\cdot)$. Since $v(\cdot)$ is concave we have that for any (s, t):

$$G(q^s)(q^t - q^s) \geqq v(q^t) - v(q^s) \tag{3.1}$$

where $G(\cdot)$ is the gradient vector of $v(\cdot)$. If the SREH holds this is equivalent to:

$$\lambda p^s(q^t - q^s) \geqq v(q^t) - v(q^s) \tag{3.1$'$}$$

If we now take any subset of indices $\{s, t, u, \ldots z\}$ we have the following inequalities:

$$\begin{gathered}
\lambda p^s(q^t - q^s) \geqq v(q^t) - v(q^s) \\
\lambda p^t(q^u - q^t) \geqq v(q^u) - v(q^t) \\
\vdots \\
\lambda p^z(q^s - q^z) \geqq v(q^s) - v(q^z).
\end{gathered}$$

Adding these, we have:

$$p^s q^t + p^t q^u + \cdots + p^z q^s \geqq p^s q^s + p^t q^t + \cdots + p^z p^z. \tag{3.2}$$

DEFINITION. *If condition* (3.2) *holds for any subset of indices then we say that the data set* $\{(p^t, q^t)\}$ satisfies cyclical monotonicity *(CM)*.

Thus we have shown that any data set that satisfies the SREH also satisfies CM. In fact, if we allow multi-valued Frisch demand functions then CM also ensures that there is some concave $v(\cdot)$ such that $\{(p^t, q^t)\}$ satisfies the SREH for that $v(\cdot)$. The equivalence of CM and the existence of a concave function $v(\cdot)$ such that $G(q^t) = p^t$ for all t is exactly Theorem 24.8 in Rockafellar (1970). A proof of sufficiency would paraphrase the proof given there. The above establishes:

PROPOSITION 1. *CM and SREH are equivalent.*

Trivially we also have that SREH2, the weaker form of the SREH, is equivalent to CM holding for adjacent years.

CM is an integrability condition. An important question is how it relates to more standard integrability conditions in economics. To answer this we consider the Generalised Axiom of Revealed Preference (GARP) given in Varian (1982). For the relation between this axiom and other revealed preference axions the reader is referred to that paper.

GARP gives the conditions under which we can rationalise a data set under the assumption of inter-temporal weak separability with constant within period preferences in each period. CM gives the conditions under which we can rationalise a data set under the assumption of inter-temporal additivity with the same utility function in each period and a single inter-temporal budget constraint. Since additivity with an unchanging utility function implies weak separability with unchanging preferences we have:

PROPOSITION 2. *CM implies GARP.*

For the sake of expositional parsimony all further proofs are deleted; interested readers can obtain proofs from the authors.

Although GARP is not a sufficient condition for CM there is a weak inverse to Proposition 2 that will be useful in our empirical work. This states that for any data set that satisfies GARP we can always find discounting factors for prices such that the data set satisfies CM.

PROPOSITION 3. *If GARP holds for* $\{(p^t, q^t)\}$ *then there exist* T *positive numbers* $\{\mu_1, \ldots, \mu_T\}$ *such that CM holds for* $\{(\mu_t p^t, q^t)\}$.

This result is significant since most aggregate time series demand data sets do satisfy GARP. This suggests that the results of tests of the SREH on such data are likely to be highly dependent on the interest rate we use to construct the discount factors used to deflate prices.

CM is a test that can be applied to any subset of goods or periods. In particular we could test CM for *s* single good over a period of time. From (3.2) we see that a necessary condition for CM is:

$$p^s q^t + p^t q^s \geqq p^s q^s + p^t q^t$$

which is more usefully written as:

$$(p^t - p^s)(q^t - q^s) \leqq 0. \tag{3.3}$$

This is a monotonicity condition; it does not generally imply CM. For the single good case, however, we do have an equivalence between (3.3) for each good alone and CM for each good alone. CM for each good is also equivalent to a stronger form of the SREH for which the within-period utility function is additive. In addition each additive component of the within-period utility function is concave. This may be useful in some contexts since to check the SREH with within-period additivity we need only draw graphs of discounted price against quantity for each good. If each graph is downwards sloping (in the sense that no two points can be joined by an upwards sloping line segment) then we cannot reject the very strong composite hypothesis of within-period additivity and the SREH. To see the force of this note that if all own real rates of interest are positive and the consumption of all goods are rising through time then all the demand curves will slope downwards. Thus we will only observe violations of CM for data with negative (positive) real rates and rising (respectively, falling) consumption in some periods. These observation are formalised in the following proposition:

PROPOSITION 4. *The following are equivalent:*

(i) *the SREH holds for some additive within-period utility function (with concave sub-utility functions for each good),*

(ii) *CM holds for each good individually,*

(iii) (3.3) *holds for each good; that is the demand curve for each good is downward sloping.*

The conditions in Proposition 4 are sufficient but not necessary for the conditions in Proposition 1. This is most clearly seen by considering (i) in Proposition 4 above which is clearly sufficient but not necessary for the SREH (see also Rockafeller 1970, end of Section 24).

We can also extend our test for CM to allow for labour supply. Let l_t be leisure in period t, L be the time endowment, $h_t = L - l_t$ be the labour supplied and w_t be the discounted money wage. If intra-temporal preferences are represented by the concave utility function $v(q^t, l_t)$ then the analogue to (3.2) is:

$$(3.4) \quad (p^s q^t - w_s h_t) + \cdots + (p^z q^s - w_z h_s) \geqq (p^s q^s - w_s h_s) + \cdots + (p^z q^z - w_z h_z)$$

To test the SREH on data that includes hours of work and wages we need to compare net expenditures ($pq - wh$) at different prices and wages rather than just expenditures as earlier. Note, however, that if $\{(p^t, q^t)\}$ and $\{(w_t, h_t)\}$ satisfy CM separately then we cannot reject the composite hypothesis for the SREH along with want independence between consumption and leisure. The labour data $\{(w_t, h_t)\}$ fails CM if and only if there are two points in the graph of (w, h) that can be jointed by a *downwards* sloping line segment, that is Frisch labour supply functions slope upwards under the SREH and within-period additivity between leisure and other goods.

All of the above has assumed interior solutions. If we have corner solutions then we do not have an equivalence between CM and the SREH. Consequently our techniques will not be applicable to micro-data with unemployment or non-purchase of some goods.

Finally, we come to two aggregation issues: aggregation over households and over goods. For the former we note that if we have many households facing the same prices and CM holds at the micro level then it holds for the aggregate data. To see this, simply add across households in the definition of CM (see also Green and Srivastava 1983). We formalise this as:

PROPOSITION 5. *If we have many households facing the same prices and CM holds for each household then CM holds for aggregate (or per capita) demands and the common prices.*

This is a very powerful aggregation theorem. No restriction is put on the form of preferences or on the heterogeneity of preferences in the population. This is in marked contrast to aggregation theorems relating to parametric demand systems (see Lau 1982, for example). Note, however, that it does not apply to labour supply data where agents face different prices (wages). In this case CM at the individual level does not imply CM for, say, average wages, hours and quantities and common prices.

For theoretical and empirical work, some aggregation across goods is forced upon us by the multiplicity of goods available to agents. At its most drastic this takes the form of assuming a single good. This is strictly only justified if preferences are homothetic or all prices move together. In empirical work the typical way we aggregate goods together is to sum, respectively, the expenditures in constant and current dollars on the goods in the commodity category to be created. The former of these sums is then taken to be the quantity; the (implicit) price (deflator) is then the ratio of the sums of current to constant dollars. If relative prices vary over time then CM at a fine level of aggregation neither implies nor is implied by CM at a coarser level of aggregation. Thus a collection of goods defined as a composite may give the opposite result in a test of CM to the result for the composite.

4. SOME RESULTS FOR CANADA, THE UK AND THE US

In this section we shall take some aggregate data for Canada, the UK and the US and see whether they satisfy GARP and CM. These data display some interesting differences and may well comprise a representative "sample." In particular the US data displays less absolute price variability but more relative price variability than the UK data.

All our time series are annual data. In principle there is no reason why we should not apply our tests to, for example, quarterly data. The problem here, however, is that we do not generally assume that preferences are constant over the year. Constancy of preferences, however, is a basic assumption for both GARP and CM. Thus we would expect to reject GARP on quarterly data. Since GARP is a necessary condition for CM this implies a rejection of CM as well.

We consider first UK per capita consumption/price data from 1952 to 1985 on seven 'goods': food, alcohol, housing, fuel, clothing, other goods and other services. These data satisfy GARP for the whole period. To check CM, we discount current prices using the "gross flat yield on 2 1/2% consols" as our nominal interest rate; the interest rate for period t is the average rate between mid-$(t-1)$ and mid-t. We shall discuss other interest rates in the next section.

Before testing CM for these goods together we consider them separately. Remember that want independence within the period and the SREH would give downwards sloping demand curves (see Proposition 4). In fact the graphs of discounted price against quantity for all of our goods have upward sloping portions. Thus we can reject the composite hypothesis of want independence and the SREH.

To test for the SREH we first test for CM for the years 1952 to 1953. Given that CM is not rejected for these two years we then test for the three years 1952–1954 and so on, until CM is rejected. Following this procedure we find that CM is not rejected for 1952–1973 but it is rejected for the period 1952–1974. We thus restart by testing for CM between 1974 and 1975. Although this procedure identifies sub-periods within which CM is not rejected this identification may not be unique. It may be, for example, that starting at 1974 and working backwards would reveal different sub-periods within each of which CM is not rejected. We report results only for the "natural" experiment of starting from the beginning of the period and working forwards.

When we apply the test for CM for the seven goods together we find that it is not rejected for the sub-periods 1952 to 1973, 1974 to 1977 and 1979 to 1985. Our results indicate that we can reconcile our data with the Strong Rational Expectations Hypothesis if we allow for shocks or surprises in 1973–74, 1977–78 and 1978–79. These can be viewed as years when revisions to "permanent" quantities were required.

Testing the weaker SREH2 on these data we find that it is only rejected for the adjacent years 1978 and 1979. Thus we find only one pair of adjacent years out of the 33 pairs between 1952 and 1985 that rejects the hypothesis that agents manage to forecast one year ahead successfully.

We also applied our test to Canadian per capita data on food, clothing, housing, house operations, medical care, transportation, recreation and other services for the period 1956 to 1981. The interest rate used to discount prices is the end of year Treasury Bill rate. These data satisfied GARP for the whole period and satisfied CM within the sub-periods 1956 to 1960, 1961 to 1969, 1970 to 1972 and 1975 to 1981.

Turning to US data on per capita consumption of food, clothing, gas, other goods, housing, house operation, transport and other services for 1954 to 1979 we find that we can accept GARP for the whole period and CM for the sub-periods 1954 to 1955, 1957 to 1973, 1974 to 1976 and 1977 to 1979.

We also consider some data on aggregate consumption and hours of work for the US. These data are similar to those used in Mankiw, Rotemberg and Summers (1985). Before testing for CM we tested for GARP and found that these data satisfy GARP for the whole period. This is in itself significant since Mankiw et al. reject their intra-temporal rationality conditions (marginal rates of substitution equal to real wage within each period) on this data set. The fact that these data do not reject GARP suggests that the latter rejection has more to do with the particular specification chosen than with the theory itself.

Turning to the test of the SREH on these data recall that want independence between leisure and consumption and the SREH would result in a downward sloping consumption graph and an upward sloping labour supply curve. We note, however, that average hours have been falling throughout most of the post-war period at the same time as (discounted) wages have been rising. Consequently the graph of hours against discounted wages is largely downward sloping so that we can reject want independence and the SREH. If, however, we apply the test for CM for labour and consumption together then we find that we cannot reject CM for the subperiods 1959 to 1972, 1973 to 1975 and 1979 to 1982. Since the graph of labour supply against discounted wage is *downward* sloping for the period 1959/72 this requires that we take leisure and goods to be (specific) complements for this period. Note that this (constant marginal utility of money) definition of complementarity is different from the Hicksian (constant utility) definition of complementarity (see Houthakker 1960). One interpretation of these results is that the strong rise in consumption over the period 1959 to 1972 is overcoming the fall in leisure that would otherwise have resulted from rising (discounted) wages.

Deaton (1985) also applies the CM test developed in this paper to adjacent years

TABLE 1

Year	Actual Rate (%)	Adjustment (%)	Inflation Rate (%)
1973/74	12.65	+1.15	17.17
1974/75	15.135	+8.3	23.7
1975/76	14.035	+1.3	15.975
1978/79	12.79	+0.1	13.48
1979/80	13.75	+3.3	17.37

of US data on aggregate consumption and hours of work. This corresponds to a test of the SREH2. Deaton finds that "violations (of CM) are more endemic than occasional." As we mentioned below Proposition 3, the outcome of CM tests will depend critically on the interest rate chosen. Deaton (1985) uses a net of tax rate that gives negative real rates of interest for many periods; we shall illustrate below how this typically biases us toward rejection of the CM condition.

Referring back to Proposition 3 we see that if a data set satisfies GARP then there are revisions to the absolute price levels in each period which ensure that the revised data set satisfies CM. The μ_t's of Proposition 3 can be considered as revisions to the discount factor R_t. The non-rejection of GARP reported above indicates that we can reconcile our data sets with the SREH by adjusting the interest rates used. We shall use the UK demand data set to illustrate; lack of space precludes an extensive examination of the other data sets considered above.

The revisions to interest rates for the UK data set were determined in the following way. The data satisfies CM for the period 1952 to 1973 but not for 1952 to 1974. We took the data for 1952 to 1974 and found the smallest change in the interest rate between 1973 and 1974 that would allow us to accept CM for this period. We then adjusted this interest rate and tested CM for 1952 onwards until we find another failure. Since this occurs in 1975 we then adjusted the interest rate between 1974 and 1975. This procedure was continued until the data with the revised interest rate series satisfied CM over the whole period. These revisions are reported in Table 1. We also report the "inflation rate" (strictly, the change in the log of the implicit price deflator for non-durables) for each year for comparison purposes. In the next section we discuss the possible significance of the fact that the "real rate" is negative for each of the years in which a revision is necessary and that the revision is always positive.

5. SOME INTERPRETATIONS OF THE UK RESULTS

In general the interpretation of the results of Afriat (1967)-Varian (1982) nonparametric tests always poses something of a problem. Conventional notions of power and significance do not sit well with yes/no type tests, although see Bronars (1987). In our discussion we shall not try for generality; rather we shall use the results of the CM test on UK data of the last section in a specific way. It is our feeling that these results are informative but the conclusions we draw are necessarily somewhat tentative.

All of the data sets looked at in Section 4 reject the SREH. This is not an unexpected result since the SREH is, in fact, a composite of (at least) three questionable assumptions: perfect foresight, inter-temporal additivity with a stable sub-utility function for each period and perfect capital markets. We have also ignored any possible inappropriate separability from other goods, any inappropriate aggregation across time or goods, rationing, errors in variables and the possibility that prices might be endogenous.

We shall not discuss any of these latter. We shall, however, examine in turn the three major component assumptions of the SREH in the light of the results presented for the UK data. We remind the reader that all of these data satisfy GARP so that we need only concern ourselves with inter-temporal considerations. We shall consequently organise our discussion around Table 1; this does not mean that we necessarily feel that we have the "wrong" interest rate (although see below). On the contrary, we believe that our results as presented in Table 1 are suggestive of rather more interesting sources of failure of the life-cycle model that are connected with the three subsidary hypotheses mentioned in the last paragraph.

Our examination will be helped by displaying the series for the marginal utility of discounted expenditure implied by the revisions in Table 1. To see how we derived this marginal utility series refer to Table 1. To reconcile our data with the SREH the discounted price for 1974 had to be discounted by $R_{73}/(1.1265 + .0115)$ rather than by $R_{73}/(1.1265)$ (and similarly for the other interest rate revisions). Thus the estimated marginal utility of discounted expenditure in 1974 (using the unrevised interest rate series) is given by:

$$\lambda_{74} = 1.1265\lambda_{73}/(1.1265 + .0115).$$

The consequent series of marginal utilities with λ_{52} set to unity is given in Table 2. The major thing to note about this series is that it is non-increasing over time. The monotonicity of the series given in Table 2 is, of course, simply a consequence of the fact that all of the adjustments in Table 1 are positive.

Perfect foresight over the period 1952 to 1985 is, perhaps, the most objectionable of our assumptions. If we weaken it to allow for uncertainty with efficient use of current information then we have the WREH given in (2.4). The series for λ observed in Table 2 is not necessarily inconsistent with the WREH, suggesting as it does that there were significant shocks in the 1970's. We could, for instance, reconcile the WREH and Table 2 by assuming a series of *upwards* revisions to (discounted) wealth which lowered the marginal utility of discounted expenditure as shown. We feel, however, that this "pleasant surprises" interpretation is inconsistent with the conventional perception of the events of the 1970's. It is also possible to give examples of "unpleasant" shocks that are consistent with this series; an obvious example is of a unanticipated rise in prices.

The wealth shocks line of reasoning given above hints at an alternative rationale for the results presented in Table 1. At each of the revision points "the real rate of interest" (the nominal rate minus the inflation rate) is negative. Under the SREH "consumption" should fall over these periods. The fact that we have to revise the nominal rate of interest upwards in each case to reconcile our data with the SREH suggests that "consumption" is not falling by enough. This is consistent with a

TABLE 2

Period	1952–73	1974	1975	1976–78	1979	1980–1985
λ_t	1	.987	.921	.911	.910	.884

ratchet model as suggested originally by Duesenberry (1949). Such a model is, of course, inconsistent with our inter-temporal additivity assumption with a *stable* sub-utility function. In this model past consumption lowers the current marginal utility of expenditure.

The monotonicity of the series in Table 2 is also exactly consistent with a particular weakening of the third assumption, the existence of perfect capital markets. Consider the extreme case where agents have perfect foresight and can lend at a fixed rate of interest but cannot borrow. In this case the marginal utility of discounted expenditure may fall from one period to the next but it will never rise. This follows since agents can always carry forward wealth but cannot re-allocate later income to earlier periods in which the marginal utility of expenditure may be high. More generally, with borrowing rates that exceed lending rates, we have that the marginal utility of discounted expenditure (using the lending rate to discount) is non-increasing (see Browning and Robb 1985).

Given the results alluded to in the previous paragraph, the hypothesis that agents have perfect foresight and unchanging utility functions but face capital markets with higher borrowing rates than lending rates is not inconsistent with the series as given in Table 2. This interpretation requires us to regard the rate of interest we have used as the lending rate and to posit that our representative agent was borrowing between the years 1973/74, 1974/75, 1975/76 and 1978/79. Although formally admissible, this interpretation is somewhat forced. In the population as a whole there are people who are neither borrowing nor lending, others who are lending or borrowing but not both and still others who are simultaneously borrowing and lending. Given this, agents face different discounted prices and the relationship between CM at the micro level and CM for the aggregate data is unclear (see the discussion following Proposition 5).

For the sake of completeness we also look at what happens if we use a different interest rate series. It is here that the CM test is most open to abuse since the results given in Table 1 indicate very clearly what sort of interest rate will lead to rejections of the SREH more often and which to rejections less often than the rate we used (that is, the consol rate). If we could find an interest rate that equals the consol rate plus the adjustments given in Table 1 then we could not, of course, reject the SREH for the whole data period. Failing such an interest rate we might note that all of the adjustments in Table 1 revise the (negative) real interest rate towards zero. Thus using the inflation rate as a nominal interest rate might well give fewer rejections of the SREH than the consol rate. Using this "interest" rate is formally equivalent to the commonly used assumption that the real rate of interest is equal to the rate of time discount (see, for example, Hall and Mishkin 1982).

Conversely we would expect that an interest rate series that is lower than the consol rate, particularly in the 1970's (the 90 day Treasury Bill rate, for example) would tend to lead to more rejections for the SREH than the consol rate.

TABLE 3

Interest Rate	Periods Within Which CM Is Not Rejected
Consol	1952–73, 74–77, 78, 79–85
Consol with 30% tax rate	1952–54, 55, 56–65, 66–70, 71, 73, 73–77, 78, 79–85
Treasury Bill	1952–54, 55–70, 71, 72–73, 74–77, 78, 79–85
Non-Durables Price Inflation	1952–73, 74–77, 78–83, 84–85

Alternatively we might assume a positive rate of time discount or a tax on capital, each of which effectively lowers the nominal interest rate used to discount prices. Table 3 presents the results for the consol rate with a constant 30% tax rate, the inflation rate and the Treasury Bill rate used as alternative rates to discount prices. We also present again the consol rate results with no utility discounting for ease of comparison.

The results here are much as we expected. The closer the interest rate to the inflation rate, the fewer the rejections of the SREH. As noted in Section 4, Deaton (1986) presents evidence for contiguous pairs of years that the SREH is decisively rejected by US data on labour supply and consumption. This investigation uses a net-of-tax interest rate and observes that condition (3.3) is not rejected so often if we have a *negative* utility discount rate. This finding is consistent with ours; generally utility levels seem to be "sticky downwards."

All of the interest rate series we have used lead to a rejection of the SREH. However, a proponent of the rational expectations hypothesis could argue that these results give a good deal of support for the hypothesis with the additional assumption that agents set the rate of time discount equal to the real interest rate or that the consol rate is the 'correct' rate. In this latter view we only need "surprises" in 1973/74, 1977/78 and 1978/79. The problem with this view is that it puts too much weight on the choice of interest rate and, more importantly, there may be other hypotheses that are also exactly consistent with these data.

As we have seen above the data is not obviously at odds with the REH if we allow for (one of) uncertainty or some intertemporal dependencies in preferences or some imperfections in capital markets. Yet another alternative is a crude Keynesian-Marshallian hypothesis which posits that total expenditure is an increasing function of disposable income and that intra-temporal allocation is made conditional on that total expenditure.

If preferences are unchanging from year to year then there are two implications of this hypothesis. First, the graph of current expenditure against current income should be upwards sloping in the sense that no two (income, expenditure) points can be joined by a downwards sloping line. The second implication is that the data should satisfy GARP. We have already seen that our data satisfy this second condition. It is also the case that the graph of total expenditures on non-durables against disposable income is upwards sloping. Thus our data are seen to be consistent with this alternative which is, of course, at considerable variance with the life-cycle hypothesis.

6. CONCLUSIONS

This paper has shown how to test data on prices, interest rates and quantities for congruence with a strong form of the rational expectations hypothesis (in fact, with a composite of some widely used assumptions). This strong form is rejected by all of the data sets we have employed although it is not rejected for (perhaps surprisingly) long sub-periods of some of them (1952 to 1973 for the UK data, for example). In the weak SREH2 form it is rejected only for one out of thirty three pairs of adjacent years for the UK data.

We have also shown that the rejections of the SREH on the UK data are consistent with a relaxation of any one of the three major components of the SREH: perfect foresight, the existence of perfect capital markets and inter-temporally additive preferences with an unchanging sub-utility function. We also show that the results of our test are critically dependent on the interest rate used.

We have also indicated that our data are consistent with at least one (Keynesian) alternative to the life-cycle hypothesis. The lesson we draw from our investigations is, in fact, that tests of alternative hypotheses on such aggregate time-series data lack power. As likely as not parametric tests of particular hypotheses on such data are simply tests of specification hypotheses on, for example, functional form or error distributions.

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CHAPTER 5

A SIMPLE NON-ADDITIVE PREFERENCE STRUCTURE FOR MODELS OF HOUSEHOLD BEHAVIOR OVER TIME

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A Simple Nonadditive Preference Structure for Models of Household Behavior over Time

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Intertemporal separability is an almost universal assumption in empirical work on household behavior, but a good deal of recent work on consumption and labor supply suggests that it may not be tenable. The traditional weakening of this assumption is to allow for habit formation. I propose an alternative structure for intertemporal preferences that nests intertemporal additivity in a simple way and yields closed-form solutions for demand functions. This structure includes the neoclassical durables model as a special case. I derive a demand system that nests the almost ideal system as its time-separable counterpart. This model is estimated on U.K. aggregate time-series data for seven goods. Time separability is decisively rejected. Moreover, I find that ignoring temporal dependencies leads to considerable bias in the estimates of elasticities. Of the seven goods, durables display the strongest nonseparable effect; the estimated reactions are consistent with the neoclassical durables model.

I. Introduction

Intertemporal separability is an almost universal assumption in empirical work on household behavior. To formulate demand systems,

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one typically invokes intertemporal weak separability so that current demands can be written as a function of current prices and current total expenditure. To develop consumption functions and labor supply equations, one typically invokes the stronger assumption of intertemporal additivity for the utility function. This widespread use suggests that intertemporal separability is a very convenient assumption. The question is, Is it justified?

That preferences may not be intertemporally separable has long been recognized. Perhaps the earliest explicit discussion appears in Marshall (1890), which relates to habits. "Irreversible" demand functions are also discussed, albeit somewhat sketchily, in Haavelmo (1944, sec. 2). This early recognition that "choices depend on tastes and tastes on past choices" (Gorman 1967) led to a number of attempts to incorporate past choices into current demands (or consumption) in empirical work (see Duesenberry 1949; Modigliani 1949; Farrell 1952; Stone 1954). Since then, the literature has grown considerably. See Phlips (1983) for references and a discussion on habits and Hayashi (1985), Novales (1985), Dunn and Singleton (1986), Muellbauer (1986), and Pashardes (1986) for further studies of consumption without the intertemporal separability assumption. See also Heckman (1979, 1981), Eichenbaum, Hansen, and Singleton (1988), Hotz, Kydland, and Sedlacek (1988), and Kennan (1988) for empirical models of labor supply or demand. What is particularly significant about these investigations is that they all reject the hypothesis that preferences are intertemporally separable.

Actually these papers are unanimous in their conclusion that household demand and supply functions display too much persistence to be consistent with intertemporally additive preferences and budgets. As Heckman (1979) notes, it is not obvious whether one can identify the source of this persistence. Browning (1988) shows that it is always possible to take demands that are generated by nonadditive preferences with additive (over time) budgets and find some other model that gives exactly these demands with additive preferences and nonadditive budgets. Sources of the latter would be imperfect capital markets in consumption studies and human capital in labor supply models. Throughout the rest of this paper I shall discuss nonseparable preferences; this is largely a stylistic convenience.

Another line of investigation that does not impose separability in looking at consumption data is presented in Davidson et al. (1978) and Anderson and Blundell (1984). The approach here is to seek good, parsimonious descriptions of the data that have only long-run congruence with theoretical constraints. Once again it seems that intertemporal effects are critical in describing the data. Parallel to these developments in the empirical literature, there has been an increasing readiness to allow for intertemporal nonseparability in preferences to account for some of the stylized facts of aggregate time series (see, e.g., Kydland and Prescott 1982).

In Section II, I present a novel preference structure for intertemporal preferences that includes additivity as a special case. I generalize intertemporal additivity by considering weaker structures on "dual" representations for preferences rather than on the primal (utility function) representation. I do this since they lead straightforwardly to corresponding structures on demand functions. I present a proposition that gives what is, in effect, the weakest generalization of additivity; I call such preferences simple nonadditive preferences (SNAP). The resultant demand functions depend on one-period lagged and one-period lead prices as well as current prices.

The SNAP structure allows that a particular good might be a substitute or a complement for itself in the periods immediately before and after the current one. To facilitate discussion of these matters, I introduce the term "autocomplementary" for a good that is a specific complement to itself in the previous period (and similarly for substitution and independence). I show that goods that are physically durable or satiating will be autosubstitutable, whereas habit-forming goods are autocomplementary. I also show that the SNAP structure includes the neoclassical durables model as a special case.

The structure in Section II is developed in the context of a perfectly certain environment. In Section III, I discuss the use of dual representations for preferences in an uncertain environment. I adopt the usual framework that agents maximize expected utility in each period and replan using all currently available information. I present some analysis that causes pessimism about the use of dual representations in such a framework when there are general intertemporal dependencies in preferences. I then go on to show that the SNAP structure developed in Section II is tractable if one is willing to assume that agents have point expectations about next-period discounted prices. Although this is a widely used assumption (usually in terms of the real rate of interest between the current period and the next period), it is a real cost that is not needed in Euler equation formulations based on the direct representation (e.g., Dunn and Singleton 1986; Hotz et al. 1988). The gain is that one can find flexible closed-form expressions for multigood demand systems. This section closes with a brief discussion of estimation strategies for intertemporal models, including the one used here.

Section IV uses the theory developed in Sections II and III to generate a demand system that nests the almost ideal demand system of Deaton and Muellbauer (1980*a*) as a special case. This generalization is reasonably parsimonious: there are as many extra parameters as goods. One feature of the formulation is that I treat all goods symmetrically in the sense that I do not make any prior presumption that any particular good is durable or habit-forming. I leave such "decisions" to the data; I regard this as an informal specification test for the approach. For example, if durables turn out not to display any intertemporal dependency but food does, then one would regard this as being evidence that something is wrong with the formulation.

In Section V, I apply the model to U.K. aggregate postwar time-series data on the demand for seven goods. These include physically durable goods such as durables and clothing as well as possibly habit-forming goods such as alcoholic beverages and tobacco and also goods that are usually thought of as not displaying intertemporal dependencies (food, e.g.). My principal finding is that intertemporal additivity is decisively rejected but that only durables and, to a limited extent, fuel display any significant autodependency over time. I also find that I cannot reject homogeneity; this substantiates to some extent the conjecture in Deaton and Muellbauer (1980*a*) that the commonly found rejection of homogeneity may be due to an inappropriate dynamic specification. I close this section with a discussion of the price and expenditure elasticities implied by the parameter estimates. I find that not taking account of intertemporal dependencies leads to considerable bias in the estimates of these, even for goods that are autoindependent.

II. A Simple Nonadditive Preference Structure

To derive the preference structure, I consider first the perfect certainty case; in the next section I show how to take account of uncertainty. The most widely used structure for the direct representation of preferences over time is the additive structure

$$U(\mathbf{q}^1, \ldots, \mathbf{q}^T) = \sum_{t=1}^{T} v^t(\mathbf{q}^t), \tag{1}$$

where $\mathbf{q}^t$ is a vector of consumptions (which are equated with purchases) of n goods in time t. The subutility functions $v^t(\cdot)$ have time superscripts to allow for, among other things, discounting and changing preferences (but note that the latter is exogenous and does not depend on past or future choices).

Considerable ingenuity has been expended in deriving empirically tractable generalizations of this structure that allow for temporal dependencies. The best known of these generalizations is the habits model of Pollak (1970); see also Houthakker and Taylor (1970), Spinnewyn (1981), and Phlips (1983). The paper by Spinnewyn, in particular, shows how one may redefine prices, quantities, and wealth so that preferences can be represented over time by an additive utility function over redefined qualities ("stocks") that is maximized subject to an additive (over time) budget constraint (see App. A for a formal account of this and the following statements on the Pollak/Spinnewyn habits model). This approach has the considerable virtue that it brings one back to familiar conceptual territory. However, the redefined quantities in period t depend on current and all past actual quantities, and the redefined prices depend on current prices and all future prices. This dependence on infinitely lagged and lead variables raises obvious problems in empirical implementation.

In practice, two variants of the Pollak-Spinnewyn model have been almost exclusively employed in empirical models; each of these solves half of the data problem discussed in the last paragraph. The first is the "habits as durables" (HAD) variant: this is the conventional neoclassical model of durables. It gives redefined prices (user costs) that are weighted sums of current and one-period lead prices. The alternative "short-memory" model (which is a special case of the structure considered by Boyer [1983]) gives redefined quantities that are a weighted sum of current and one-period lagged quantities. To give two examples: Eichenbaum et al. (1988) estimate a model of consumption and labor supply in which consumption is always short-memory and leisure is either short-memory or HAD. Dunn and Singleton (1986) estimate a system for durables and nondurables in which the former is taken (not surprisingly) to have a HAD structure while the latter is modeled as a short-memory.

I present here a rather different approach that addresses three criteria directly. First, I wish to find a preference structure that nests additivity over time in a simple way. This will facilitate testing of the additivity hypothesis. Second, I shall be explicitly concerned with parsimony in data from the outset. That is, I shall avoid, as far as possible, having to take consideration of variables indefinitely far back or forward. Third, I shall work with dual representations for preferences (i.e., functions defined on prices and some measure of welfare). Such representations have proved particularly fruitful in empirical allocation models that assume intertemporal separability (see, e.g., Deaton and Muellbauer 1980*b*), and it is natural to try to extend them to the more general case. As an example, it is now well established that the almost ideal demand system of Deaton and Muellbauer (1980*a*) provides a good approximation to many demand data sets, but these preferences do not have a closed-form direct utility function.

There are a number of dual representations that one could begin with. I choose to work with the profit function representation since this is the only representation that "inherits" additivity from the direct utility function. The profit function representation is defined by

$$\pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r) = \max_q \left[rU(\mathbf{q}^1, \ldots, \mathbf{q}^T) - \sum_t \mathbf{p}^t \mathbf{q}^t \right], \tag{2}$$

where $\mathbf{p}^t$ is a vector of prices in period t discounted to the first period and $\mathbf{p}^t\mathbf{q}^t$ is an inner product; the interpretation of r is given in the next paragraph. In this formulation, I am implicitly assuming the existence of a perfect capital market with a single rate of interest. The profit function is well known from producer theory (see McFadden 1978); it was first suggested as a representation for consumer preferences in Gorman (1976). The profit function is convex and linear homogeneous in $(\mathbf{p}^1, \ldots, \mathbf{p}^T, r)$; further discussion can be found in Browning, Deaton, and Irish (1985).

The first-order condition for the maximization problem on the right-hand side of (2) gives an immediate interpretation of r as the inverse of the marginal utility of expenditure (i.e., the marginal cost of utility). Applying the envelope theorem to (2), we can derive the Frisch (or constant marginal utility of expenditure) demand functions in period t:

$$\mathbf{q}^t = -\nabla_t \pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r), \tag{3}$$

where ∇_t denotes the gradient of π with respect to $\mathbf{p}^t$. Frisch demands are zero homogeneous in all prices and r and are symmetric in the sense that

$$\frac{\delta q_i^t}{\delta p_j^s} = \frac{\delta q_j^s}{\delta p_i^t}, \quad \text{for all } i, j, t, s. \tag{4}$$

This symmetry condition is an integrability requirement; it ensures that there is a preference ordering behind the Frisch demands. This is the defining characteristic of rational allocation with intertemporal dependencies: if the past affects the present, then rational agents will take account of present actions on future preferences.

Frisch demand functions have been much used in recent years. They first appeared in the empirical literature in the habits models of Houthakker and Taylor (1970) and Phlips (1983). This is no coincidence. As Heckman (1974) observes, explicit consideration of intertemporal allocation leads naturally to the use of Frisch demand functions. The reason is that such demand functions take the marginal utility of (discounted) wealth as fixed, and this is what rational agents themselves are trying to equate over time.

If $U(\cdot)$ takes the form given in (1), then the profit function defined in (2) is also additive:

$$\pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r) = \sum_t \hat{\pi}^t(\mathbf{p}^t, r),$$

and Frisch demands depend only on current prices and r:

$$\mathbf{q}^t = -\nabla_t \hat{\pi}^t(\mathbf{p}^t, r). \tag{5}$$

In this case we have want independence for goods in different years: $\delta q_i^t/\delta p_j^s = 0$ for $t \neq s$. This property of Frisch demands is necessary for the intertemporal additivity assumed in (1). Given this, we see that any demand system that does not have this property must be derived from nonadditive preferences. Perhaps the simplest weakening of this is to allow current Frisch demands to depend only on past and current prices:

$$\mathbf{q}^t = f^t(\mathbf{p}^t, \mathbf{p}^{t-1}, r). \tag{6}$$

With (4), however, it is trivial to show that this structure and rationality are equivalent to additivity. Essentially, (6) is a "myopic" structure: no account of the future is taken when making current decisions (beyond those working through the budget constraint).

Following this line, however, does give a simple generalization of intertemporal additivity:

$$\mathbf{q}^t = g^t(\mathbf{p}^{t-1}, \mathbf{p}^t, \mathbf{p}^{t+1}, r). \tag{7}$$

The following proposition establishes the structure of preferences that gives this form (proofs of all propositions are given in App. B).

PROPOSITION 1. Frisch demands take the form given in (7) if and only if

$$\pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r) = -\sum_{t=1}^{T-1} \Phi^t(\mathbf{p}^t, \mathbf{p}^{t+1}, r).$$

I shall refer to this structure as a SNAP structure.[1] Each $\Phi(\cdot)$ function is concave and linear homogeneous in $(\mathbf{p}^t, \mathbf{p}^{t+1}, r)$ and increasing in $(\mathbf{p}^t, \mathbf{p}^{t+1})$; in fact each is itself a "loss" function (i.e., the negative of a profit function).

Applying (3), we see that the imposition of a SNAP structure gives a specific form to Frisch demands, as stated in the following corollary.

[1] Note that a function has a SNAP structure if and only if the Hessian of the function has a (block) tridiagonal structure. An $nT \times nT$ matrix is (block) tridiagonal if each $n \times n$ block on the diagonal and on each side of the diagonal is nonzero and all other elements are zero (see Graybill 1969).

COROLLARY 1. The Frisch demands for a SNAP structure are of the form

$$\mathbf{q}^t = \nabla_t \Phi^{t-1}(\mathbf{p}^{t-1}, \mathbf{p}^t, r) + \nabla_t \Phi^t(\mathbf{p}^t, \mathbf{p}^{t+1}, r).$$

Thus current demands can be thought of as being the sum of two components: a current demand that myopically takes account of the past and a current demand that ignores the past but takes account of the effect of current actions on future preferences.

Before we move on to allocation under uncertainty, it may be useful to give some of the implications of assuming a SNAP structure since it is a new form. First, note that if we have a SNAP structure, then total expenditure in period t is given by

$$x_t = \sum_i p_i^t q_i^t = \sum_i p_i^t g_i^t(\mathbf{p}^{t-1}, \mathbf{p}^t, \mathbf{p}^{t+1}, r). \tag{8}$$

Assume now that total expenditure in period t is "normal" in the sense that, for given prices, it is an increasing function of lifetime wealth. Also, if preferences are such that the marginal utility of expenditure ($= r^{-1}$) is strictly decreasing in wealth (essentially, if the lifetime utility function is strictly concave), then x_t is a strictly monotone function of r in (8). Thus we can invert on r in the equation above and substitute back in (7). This establishes the following corollary.

COROLLARY 2. If we have a SNAP structure and total expenditure in each period is normal, then Marshallian demands take the form

$$\mathbf{q}^t = h^t(\mathbf{p}^{t-1}, \mathbf{p}^t, \mathbf{p}^{t+1}, x_t). \tag{9}$$

This form for Marshallian demands is satisfyingly simple in its temporal structure.[2] It will form the basis for our estimates below.

We can also relate our structure to the more familiar Pollak-Spinnewyn habits structure discussed above.

PROPOSITION 2. Preferences have a SNAP and Pollak-Spinnewyn structure if and only if they have a HAD structure.

To see this informally, note that if we have a HAD model, then we can reformulate each $\Phi^t(\cdot)$ in proposition 1 in the following way:

$$\Phi^t(\mathbf{p}^t, \mathbf{p}^{t+1}, r) = \hat{\Phi}^t(\mathbf{p}^t - \mathbf{p}^{t+1}(\mathbf{I} - \mathbf{M}), r), \tag{10}$$

where $\mathbf{M}$ is a diagonal "memory" matrix with diagonal element unity for a nondurable good (see App. A). Thus current purchases depend

[2] Note, however, that this is only necessary and not sufficient for SNAP. To see this, consider preferences that are intertemporally separable (but not additive). In this case $\mathbf{q}^t$ depends only on $(\mathbf{p}^t, x_t)$ so that (8) holds, but preferences need not take a SNAP form.

on lagged and current user costs. These determine, respectively, the stock carried over from period $t - 1$ and the stock passed on to period $t + 1$. This in turn determines the purchases in the current period. Note that SNAP generalizes the neoclassical durables model since most functions $\Phi(\cdot)$ cannot be written in the form $\hat{\Phi}(\cdot)$ in (10).

Generally, we can classify goods i and j as specific complements, specific substitutes, or want-independent according to whether the cross-partial of the (Frisch) demand for good i with respect to the price of good j is negative, positive, or zero, respectively (see Houthakker 1960). Our proposed structure has the property that all goods are want-independent of all other goods in nonadjacent periods.[3]

To interpret the SNAP structure further, consider a perfect-foresight, one-good model.[4] To analyze the effect of an anticipated and permanent change in the price of the good, suppose that up to time t the price is p and thereafter it is $p + \Delta p$ (where Δp is small). To a first-order approximation, the first differences in demands are given by

$$\Delta q^{t-1} = \Phi_{12}\Delta p, \tag{11}$$

$$\Delta q^{t} = (\Phi_{22} + \Phi_{11})\Delta p, \tag{12}$$

and

$$\Delta q^{t+1} = \Phi_{21}\Delta p, \tag{13}$$

where the ij subscript on the Φ's refers to partial derivatives, all evaluated at (p, p, r), and Δ is the first-difference operator. By concavity, Φ_{ii} is negative so that (12) implies that the within-period (Frisch) response to a price rise is negative.

Turning to (11) and (13), note first that these two responses are equal. This is an implication of symmetry and the structure given in corollary 1. This is a strong prediction that can, potentially, be rejected by the data. If Φ_{12} is negative, then the good is a (specific) complement for itself in the previous and next periods. I shall refer to such a good as *autocomplementary*. Similarly, a good is *autosubstitutable* (*autoindependent*) if Φ_{12} is positive (zero). It is easy to show that in a neoclassical durables model (see [10] above), durable goods are autosubstitutable and habit-forming goods are autocomplementary. Even if a particular good is not physically durable, it might exhibit autosubstitutability if, for instance, it is satiating in the sense that high consumption last period lowers the current marginal utility of the good.

[3] That is, $\delta q_i^t / p_j^s = 0$ for all $s \neq t - 1, t$, and $t + 1$.

[4] This is for convenience only; it means that we can drop the i subscript.

We can also see that the long-run response to a permanent change is nonpositive since

$$\Delta q^{t-1} + \Delta q^{t} + \Delta q^{t+1} = (\Phi_{11} + \Phi_{22} + 2\Phi_{12})\Delta p, \tag{14}$$

which is negative if Δp is positive since $\Phi(\cdot)$ is concave.

III. Allocation in an Uncertain Environment

In this section I consider the problem of modeling decision making under uncertainty using dual representations for preferences. I shall argue that in the general case of nonadditive preferences we cannot do this without invoking unacceptably strong assumptions such as perfect foresight but that we can do it if we assume SNAP. The environment I have in mind is the familiar one in which an agent receives new information each period and chooses a current vector of purchases to maximize expected utility using all the currently available information. Once again we shall deal with the one-good case.

In period t the agent has expected utility $E_tU(\bar{q}^1, \ldots, \bar{q}^{t-1}, q^t, \tilde{q}^{t+1}, \ldots, \tilde{q}^T)$, where $E_t(\cdot)$ is the expectations operator conditional on the information set at time t. The quantities chosen in the past are $(\bar{q}^1, \ldots, \bar{q}^{t-1})$. The notation $\tilde{z}$ is used to denote that z is a random variable. The agent chooses q^t to maximize this subject to the budget constraint holding in each state of the world (see, e.g., Epstein 1975). The familiar Euler equation is

$$E_t\left(\frac{\tilde{U}_t}{p^t}\right) = E_t\left(\frac{\tilde{U}_{t+1}}{\tilde{p}^{t+1}}\right), \tag{15}$$

where the subscripts on U refer to partials. Note that all the prices here are discounted. A more familiar way to write this equation is

$$E_t(\tilde{U}_t) = E_t[(1 + \tilde{b}_t)\tilde{U}_{t+1}], \tag{15'}$$

where $\tilde{b}_t$ is the (random) real rate of interest between periods t and $t + 1$.

It is important to note that if preferences are not additive over time, then U_t is not known at time t (since it depends on future random quantities). Hence we can talk only of the expected marginal utility of discounted lifetime wealth in period t ($= E_t[\tilde{U}_t/p^t]$).

Given the utility function $U(\cdot)$, we can define a restricted profit function representation

$$\begin{aligned} &\phi^t(p^1, \ldots, p^{t-1}, q, p^{t+1}, \ldots, p^T, r_t) \\ &= \max\left[r_tU(q^1, \ldots, q^{t-1}, q, q^{t+1}, \ldots, q^T) - \sum_{s \neq t} p^s q^s\right], \end{aligned} \tag{16}$$

where the maximization is over $(q^1, \ldots, q^{t-1}, q^{t+1}, \ldots, q^T)$. This is just another way of describing the preference over certain outcomes that are represented by $U(\cdot)$. There is, as yet, no presumption that the lagged prices in $\phi(\cdot)$ are those that actually prevailed in the past nor that the lead prices are among those that are possible in the future. We shall interpret r_t below. From the envelope theorem and (16), we have

$$\phi_t^t = r_t U_t \tag{17}$$

and

$$\phi_{t-s}^t = -q^{t-s}, \quad s = 1, 2, \ldots, t-1. \tag{18}$$

This restricted profit function is related to the profit function defined in (2) by

$$\pi^t(p^1, \ldots, p^{t-1}, p^t, \tilde{p}^{t+1}, \ldots, \tilde{p}^T, r_t) = \max_{q^\cdot} \{E_t \phi^t - p^t q\}. \tag{19}$$

The first-order conditions for this maximization are

$$E_t \phi_t^t = p^t. \tag{20}$$

Combining (17) and (20), we have $r_t^{-1} = E_t(U_t/p^t)$. Thus the interpretation of r as the inverse of the marginal utility of wealth in the perfect certainty case carries over for r_t in the uncertainty case, except that now we must talk of the expected marginal utility (conditional on information in time t). Note that this expected value is, of course, known in period t, although the actual value may not be known until later.

The representation in (19) is potentially useful since, by the envelope theorem, current demands can be derived by differentiation:

$$q^t = -\pi_t^t. \tag{21}$$

To be actually useful, however, we need to deal with three issues. First, r_t is not observable. This is a familiar problem; we shall postpone discussion of it until the end of this section. The second problem with using (19) is that the demands defined by (21) depend on all future (random) prices. Imposing SNAP helps considerably with this since now q^t depends explicitly only on one-period-ahead prices; the effects of all other future prices are "rolled up" in r_t. We shall assume that agents have point expectations over next-period discounted prices; let these be given by ${}^a p^{t+1}$, where the a superscript denotes "anticipated."[5] We shall discuss this assumption further in the empirical section below.

[5] This replacement of a nondegenerate density by a degenerate one is strictly valid only if π^{*t} is affine in some known function of lead prices. If this is the case, then when we take conditional expectations in (19), we are left with an affine function of the conditional expectation of the function of lead prices. These conditional expectations would then be genuine "anticipated" variables.

The third problem with using (19) can be seen by noting that applying the envelope theorem to (19) and using (18) imply

$$\pi^t_{t-s} = E_t \phi^t_{t-s} = -q^{t-s}, \quad s = 1, 2, \ldots, t-1. \tag{22}$$

To ensure that we are taking correct account of past decisions on current preferences, we need to choose lagged "virtual" prices so that

$$\pi^t_{t-s} = -\bar{q}^{t-s}, \quad s = 1, 2, \ldots, t-1. \tag{22'}$$

In general there is nothing to ensure that these virtual prices will be equal to the actual values of past discounted prices. It is this that makes us pessimistic about using dual methods in the general analysis of allocation under uncertainty since these virtual prices are inherently unobservable.[6]

I shall now show how imposing SNAP and a regularity condition allows us to circumvent this problem. First I shall show that given SNAP (and our regularity condition) for any actual (p^t, r_t), there is a unique set of virtual prices $(\hat{p}^1, \ldots, \hat{p}^{t-2}, p^{t-1})$, where the one-period lagged prices are the actual prices observed, such that (22′) is satisfied. Next observe that although the lagged two-period and earlier virtual prices so defined are unobservable, they do not affect current demands since, by SNAP, these depend on only one-period lagged and lead prices, current prices, and r_t.

If (19) has a SNAP structure, then (by corollary 1) the two-period lagged virtual price is defined by the implicit relationship

$$\bar{q}^{t-1} = \Phi_2(\hat{p}^{t-2}, p^{t-1}, r_t) + \Phi_1(p^{t-1}, p^t, r_t), \tag{23}$$

where, as before, a subscript on a function denotes a partial. Of course, this is not an actual demand function; rather it is an implicit inverse demand function. If Φ_{21} is nonzero, then we can invert this relationship and define $\hat{p}^{t-2}$ as a function of the actual q^{t-1} chosen (i.e., $\bar{q}^{t-1}$) and the actual (p^{t-1}, p^t, r_t). If, on the other hand, this cross-partial is zero, then we are back with the additive model. Thus our regularity condition is that if a good is autosubstitutable (autocomplementary) for some set of prices, then it is autosubstitutable (autocomplementary) for all prices in our domain. Given this, the virtual price defined by (23) is unique. We can now go on to define $\hat{p}^{t-s}$ for $s = 2, 3, \ldots$ by the recursive implicit relationship

$$\bar{q}^{t-s} = \Phi_2(\hat{p}^{t-s-1}, \hat{p}^{t-s}, r_t) + \Phi_1(\hat{p}^{t-s}, \hat{p}^{t-s+1}, r_t).$$

[6] This suggests that the best modeling strategy would be to define a preference representation on lagged quantities and current and lead prices (yet another restricted profit function!). Unfortunately, it can be shown that the demands generated in this way must depend either on all future prices or on all past quantities (or be additive). That is, truncating either one of the infinite lagged or lead series precludes truncating the other.

If we now assume SNAP and our regularity condition and take the unique virtual prices defined above, then from (21) we have

$$q^t = \Phi_2(p^{t-1}, p^t, r_t) + \Phi_1(p^t, {}^a p^{t+1}, r_t). \tag{24}$$

Thus if we parameterize $\Phi(\cdot)$, we can write current demands as a closed form of lagged and current prices, lead anticipated prices, and the expected marginal utility of wealth. I end this section with a discussion of ways of dealing with the unobservability of the latter.

There are three broad schemes for dealing with the fact that r_t is not observed by the econometrician. First, one can use the Euler equation itself:

$$E_t(r_t^{-1}) = E_t(r_{t+1}^{-1}). \tag{25}$$

If we parameterize $\Phi(\cdot)$ so that some known function of q^t in (24) is additive in some function of r_t^{-1}, then we can use the conventional generalized method of moments technique to estimate the parameters of $\Phi(\cdot)$.[7] Although much used in recent years, this approach is not without problems. First, one has to maintain the orthogonality conditions implicit in (25). Ideally one would like to test these, particularly since tests for any overidentifying orthogonality conditions usually reject these conditions. Second, it has proved difficult to allow for measurement errors and taste shocks in such models.

A second approach is to treat first differences in r_t^{-1} as a genuine latent variable. In Attfield and Browning (1985), it is shown that if we have a time-separable model and impose a subset of the intratemporal integrability conditions (homogeneity and symmetry) on a model with at least three goods in each period, then we can identify all the preference parameters in a Frisch system. This then allows us to test all the orthogonality conditions of the first approach. The cost of this approach is that we need to maintain integrability conditions that are usually rejected in demand studies on aggregate data. This brings us to the final estimation strategy.

The derivation of corollary 2 above showed how we can go from a system conditioned on prices and the marginal utility of wealth (a Frisch system) to one conditioned on prices and current total expenditure (a Marshallian system). In the latter, all the variables in the

[7] Starting with the direct utility function, it has been possible to ensure this additivity in forecast errors only by making strong assumptions on preferences. For example, the preferences used by Dunn and Singleton (1986) are defined on the service flows from durables and nondurables purchases. The within-period utility function on these flows is taken to be Cobb-Douglas and hence implies that within-period preferences are homothetic and additive. As another example, Hotz et al. (1988) use a more flexible utility function. As they admit, this does not lead to an additive forecast error, so they have to resort to an approximation. In contrast, it has proved possible to find less restrictive forms that give additivity in forecast errors starting from the profit function, at least for the time-additive case (see Browning et al. 1985).

system are observable.[8] The cost of this approach is that not all the parameters of the Frisch system can be identified from the Marshallian system; essentially the mapping from Frisch to Marshallian is many-one. This is not surprising; we cannot hope to identify all the parameters governing intertemporal allocation from observing only intratemporal allocation. On the other hand, we do not need the strong identifying assumptions implicit in the first two methods discussed above. In the empirical work I shall adopt this final approach. Thus one trades off generality for robustness; I shall discuss this further once a parameterization for preferences has been adopted. It is to this that I now turn.

IV. A Nonadditive Demand System

In this section I develop a demand system for many goods that allows for intertemporal dependencies in preferences. As stated at the end of the last section, this will be estimated by conditioning on total expenditure so that it is natural to start with preferences that are known to fit the data reasonably well. I choose to work with the almost ideal demand system of Deaton and Muellbauer (1980*a*). The procedure is as follows. First, I shall find the profit function for these preferences under the assumption of intertemporal additivity. Taking the function for a single period, I shall then add lagged prices to derive the $\Phi(\cdot)$ function in proposition 1. Finally, I derive the Marshallian budget shares for good i by using (8). Below I sketch only the details of this derivation.

Let us start with intertemporally additive preference and assume that preferences in any period can be represented by the PIGLOG cost function:

$$\ln c(\mathbf{p}, u) = \ln a(\mathbf{p}) + ub(\mathbf{p}), \tag{26}$$

where $a(\cdot)$ is linear homogeneous and $b(\cdot)$ is zero homogeneous. The profit function associated with this is

$$\pi(\mathbf{p}, r) = \left\{\ln\left[\frac{r}{b(\mathbf{p})}\right] - \ln a(\mathbf{p}) - 1\right\}\frac{r}{b(\mathbf{p})}. \tag{27}$$

It is critical to note that in (26) we have taken a particular normalization for the within-period utility function: if we replaced u by $F(u)$, where $F(\cdot)$ is strictly increasing, then we would have a different profit

[8] The great majority of demand studies can thus be thought of as being life cycle consistent in this sense as well as, e.g., systems of labor supplies and commodity demands as in Blundell and Walker (1986).

function. We shall return to the implications of this choice for intertemporal allocation below.

Let us now introduce lagged prices into (27) to break intertemporal separability. Let

$$\Phi(\mathbf{p}^t, \mathbf{p}^{t-1}, r) = -\left\{\ln\left[\frac{r}{b(\mathbf{p}^t)}\right] - \ln a(\mathbf{p}^t) - 1 + \ln d(\mathbf{p}^{t-1})\right\}\frac{r}{b(\mathbf{p}^t)}, \tag{28}$$

where $d(\cdot)$ is a zero homogeneous function. To derive the demands in period t, we have

$$\mathbf{q}^t = \nabla_t\Phi(\mathbf{p}^t, \mathbf{p}^{t-1}, r_t) + \nabla_t\Phi(\mathbf{p}^{t+1}, \mathbf{p}^t, r_t), \tag{29}$$

where, note, we have dated r as required in (24).

If we now use (8), we can show that

$$x_t = \frac{r_t}{b(\mathbf{p}^t)} \tag{30}$$

and use this relationship to substitute for the unobservable r_t. This relationship makes explicit the dependence of total expenditure on current prices; I shall take account of this endogeneity in the empirical work below. Returning to the issue of the choice of normalization in (26), note first that all the values in (30) are stated in discounted terms. If there is an anticipated change in the nominal interest rate (or, the same thing, a uniform change in all prices), then r_t is held constant (since the change is anticipated) and $b(\mathbf{p}^t)$ does not change (since it is zero homogeneous). Thus x_t is unaffected by such a change. This means that by assuming the normalization given in (26), we are constraining the intertemporal substitution elasticity to be minus one; that is, a 1 percent rise in the (discounted) price of consumption (which is well defined since all prices move together) leads to no change in expenditure and a consequent 1 percent fall in consumption. As discussed at the end of the last section, this is the parameter that we cannot identify from a conditional demand system. We shall return to this issue yet again below.

In estimation we adopt the following parameterizations:

$$\ln a(\mathbf{p}) = \sum_k \alpha_k \ln p_k + \tfrac{1}{2}\sum_k \sum_l \mu_{kl} \ln p_k \ln p_l, \tag{31}$$

$$\ln b(\mathbf{p}) = \sum_k \beta_k \ln p_k, \tag{32}$$

and

$$\ln d(\mathbf{p}) = \sum_k \theta_k \ln p_k, \tag{33}$$

where $\Sigma_k\ \alpha_k = 1$, $\Sigma_k\ \beta_k = \Sigma_k\ \theta_k = \Sigma_k\ \mu_{1k} = 0$, and $\mu_{k1} = \mu_{1k}$. Combining (28)–(33) gives the following budget share function for good i in period t:

$$\begin{aligned} w_{it} = \alpha_i + \sum_k \mu_{ik} \ln p_k^t + \beta_i \ln\left[\frac{x_t}{a(\mathbf{p}^t)}\right] \\ + \beta_i \sum_k \theta_k \ln p_k^{t-1} - \theta_i\left[\frac{b(\mathbf{p}^t)}{b(\mathbf{p}^{t+1})}\right]. \end{aligned} \tag{34}$$

This is a conventional almost ideal demand system except for the last two terms, which involve only as many parameters for the system as a whole as there are goods.[9] Intertemporal separability is equivalent to all the θ's being zero. One feature of this formulation that is important to note is that since the β's and θ's each sum to zero, only relative prices matter in the indices $\ln d(\mathbf{p}^{t-1})$ and $b(\mathbf{p}^t)/b(\mathbf{p}^{t+1})$. Thus only relative price changes from last period to the current one and from the current period to the next period matter.

We now derive some of the implications of these parameters. To find the elasticities for a permanent and anticipated price change of Δp_i in the price of good i in period t as in (11)–(13), we use (29)–(33), holding r constant:

$$\frac{\Delta \ln q_i^{t-1}}{\Delta \ln p_i} = \frac{\beta_i \theta_i b(\mathbf{p}^{t-1})}{w_{it-1} b(\mathbf{p}^t)}, \tag{35}$$

$$\frac{\Delta \ln q_i^t}{\Delta \ln p_i} = \frac{\mu_{ii} - \beta_i(\alpha_i + \theta_i)}{w_{it}} + \left(1 + \frac{\beta_i}{w_{it}}\right)\left(\frac{\Delta \ln x_t}{\Delta \ln p_i^t}\right) - 1, \tag{36}$$

$$\frac{\Delta \ln q_i^{t+1}}{\Delta \ln p_i} = \frac{\beta_i \theta_i}{w_{it}} + 1. \tag{37}$$

[9] The almost ideal system does have one theoretical drawback (which is common to most other budget share systems): if deflated expenditure trends upward indefinitely, then for goods with $\beta_i > 0$ (< 0) we shall eventually end up predicting budget shares greater than unity (less than zero). There are two responses to this. We could choose an alternative specification for which we can have nonhomothetic preferences that do not eventually violate the nonnegativity constraints (see, e.g., Lewbel 1988). Alternatively, we could allow for the censoring of the budget shares in the system we use. On micro data this might be necessary (see Van Soest and Kooreman [1990] for a discussion and references). On aggregate time-series data, it is clear that since all the budget shares are a long way from their boundaries in probability terms, we can safely ignore the censoring.

Taking (36) first, we see that the within-period response to an anticipated and permanent price rise depends on the estimated parameters and on how much the agent changes total expenditure consequent on the change. This latter is governed by the intertemporal substitution elasticity, which is not estimated in a Marshallian system.

Although $\Delta \ln x_t/\Delta \ln p_i^t$ is not estimated, we can derive a value for it if we assume a value for the intertemporal substitution elasticity (see Browning [1989] for a derivation of the intertemporal substitution elasticity in the many-good case). In this case, coherence suggests taking the consumption function given in (30) for which the elasticity is -1. For this function, $\Delta \ln x_t/\Delta \ln p_i^t$ is equal to $-\beta_i$. Alternatively, if we set the change in total expenditure equal to zero, then we have the conventional Marshallian or uncompensated own-price elasticity.

The relationships (35) and (37) determine whether a particular good is autocomplementary, autosubstitutable, or autoindependent. Note that since these have the same sign as $\beta_i\theta_i$, the regularity condition defined in the last section is satisfied for these preferences. As can be seen, a zero value for θ_i is sufficient but not necessary for good i to be autoindependent.

Finally, note that the conventional expenditure elasticity is given by

$$\frac{\Delta \ln q_i^t}{\Delta \ln x_t} = 1 + \frac{\beta_i}{w_{it}}. \tag{38}$$

Thus good i is a luxury if and only if β_i is greater than zero. This brings out another feature of the specification: if we take the consumption function (30) as governing intertemporal allocation, then a rise in the price of good i causes total expenditure to rise if and only if good i is a necessity (see the last paragraph).

V. Results on U.K. Time-Series Data

I estimate the model on U.K. aggregate annual time-series data from 1954 to 1985 (1953 and 1986 are used to create lags and leads); these data are taken from the Central Statistical Office (1988). I model seven goods that cover all expenditures except for those on housing; the latter are excluded since I considered them too heterogeneous to be usable. A list of the goods modeled can be found in table 1. All seven goods are, of course, wide composites, and most include items that are physically durable. As examples (beyond the obvious ones of durables and clothing), note that "other goods" includes cameras, jewelry, and toys; "fuel and lighting" includes coal and other storable fuels. Also, many of the component goods may be habit-forming or

TABLE 1

SOME DATA

	BUDGET SHARE (%)		LOG PRICE (1980 = 0)	
GOOD	1954 (1)	1985 (2)	1954 (3)	1985 (4)
Durables	7.9	11.2	−1.43	.15
Food	29.0	16.1	−1.70	.26
Alcohol and tobacco	14.5	12.2	−1.63	.48
Clothing and footwear	10.6	8.0	−1.29	.19
Fuel and lighting	5.0	10.0	−2.17	.45
Other goods	10.0	12.1	−1.68	.31
Services	23.0	30.5	−2.04	.37

satiable. Apart from the obvious examples of alcohol and tobacco, one cannot rule out, a priori, that other less obvious things (such as telephone charges, which are the largest item in services) are also subject to "ratchet" effects. All of this means that I see no reason to restrict temporal dependencies to a prespecified collection of goods.

Table 1 also presents some of the data. As can be seen, there have been some substantial changes in the structure of expenditures over the 32 years of the sample. The most important of these are that the budget shares of fuel and services have increased by 100 percent and 33 percent, respectively, while the food share has fallen by 44 percent. At the same time, there have also been large changes in relative prices. As can be seen from columns 3 and 4, all prices have increased in absolute level, but the range has been quite wide. Thus clothing and food (log) prices increased by 1.48 and 1.58, while the (log) prices of services and fuel rose by 2.41 and 2.62, respectively. This translates into about an annual 2 percent relative price change between clothing and fuel. Finally, note that real consumption rose by 112 percent over the sample period. The purpose of a demand system is to bring these facts together in a coherent way; it is to this that I now turn.

Before I discuss the stochastic specification for the model, I present some statistics for an almost ideal system with no account taken of intertemporal dependencies (see table 2). This is (34) without the last two terms and with an error term added. I have followed previous investigators and replaced the $a(\mathbf{p}^t)$ deflator by a budget share–weighted Stone price index P_t (see Deaton and Muellbauer 1980*a*). With this approximation, ordinary least squares is equivalent to full information maximum likelihood; the estimates given are the ordinary least squares estimates. Thus no account is taken here of the possible endogeneity of deflated total expenditure; in the SNAP system below I shall take account of this.

As can be seen, all the budget share equations fit quite well, but

TABLE 2

ALMOST IDEAL SYSTEM

Good	R^2	Durbin-Watson Statistic	t-Test for Homogeneity
Durables	.86	1.41	−.54
Food	1.00	1.80	−2.65
Alcohol	.91	1.18	.25
Clothing	.98	1.05	−.07
Fuel	.97	.94	.22
Other goods	.96	1.66	.41
Services	.99	1.43	1.88

many exhibit some sign of dynamic misspecification. Also, note that homogeneity (i.e., $\Sigma_k \mu_{ik} = 0$) is rejected for food. On the other hand, these results are rather better than those usually seen for applications of the almost ideal system to aggregate data. Typically when durables are not included in the set of goods modeled, homogeneity is rejected for many goods, and many equations have an R^2 that is greater than the Durbin-Watson statistic. It seems that adding durables to the system clears up some problems even if one takes no account of durability. Although this is an improvement, there are still clear signs of problems in the results presented in table 2.

Many (nonexclusive) reasons have been advanced for the failure of the simple model. For example, it might be that we simply have the wrong functional form for preferences. This is strongly supported by the fact that most aggregate demand data (including the data used here) satisfy revealed preference conditions that ensure that there is a utility function that rationalizes the observed data exactly (see Varian 1982). An alternative reason for the failure is that we are aggregating across households inappropriately. As the results of Stoker (1986) indicate, this will typically lead to bias in our parameter estimates and will induce serial correlation in the errors. A third reason may be that we are imposing the wrong temporal preference structure on the data; it is to testing this that I now turn.

I shall begin by rewriting (34) in a slightly more general way and by including an additive error term. This error term is intended to capture taste shifts, measurement error in the dependent variable, and the effects of left-out variables. Thus we have

$$\begin{aligned} w_{it} = \alpha_i + \sum_k \mu_{ik} \ln p_k^t + \beta_i \ln y_t + \beta_i \sum_k \theta_k \ln p_k^{t-1} \\ + \delta_i \left[\frac{b(\mathbf{p}^t)}{b(\mathbf{p}^{t+1})} \right] + e_{it}, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{39}$$

where y_t is deflated total expenditure ($= x_t/P_t \approx x_t/a(\mathbf{p}^t)$). This differs from (34) by the relaxation that the coefficients on the "lead" index (i.e., the δ's) are not necessarily the same as the coefficients on the lagged index (i.e., the θ's). The restrictions from the SNAP model are

$$\text{current homogeneity:} \quad \sum_k \mu_{ik} = 0 \quad \text{for all } i, \tag{40}$$

$$\text{lagged homogeneity:} \quad \sum_k \theta_k = 0, \tag{41}$$

$$\text{intratemporal symmetry:} \quad \mu_{ij} = \mu_{ji} \quad \text{for all } i,\ j, \tag{42}$$

$$\text{intertemporal symmetry:} \quad \theta_i = -\delta_i \quad \text{for all } i. \tag{43}$$

Of these, (43) is of particular interest to us: it embodies the constraint that the changes in the periods immediately before and after an anticipated price change are equal; see (11) and (13) and (35) and (37). One other restriction that we are obviously interested in is

$$\text{no SNAP:} \quad \theta_i = \delta_i = 0 \quad \text{for all } i. \tag{44}$$

This is, of course, the test for intertemporal additivity.

In a system without (43), we can also test for myopic behavior since the effect of future variables can be set to zero without simultaneously setting the lagged index to zero. Thus we have another test for

$$\text{myopic behavior:} \quad \delta_i = 0 \quad \text{for all } i. \tag{45}$$

Since this restriction and SNAP together imply additivity (i.e., [44]), it is not nested in the SNAP system, so that in our testing below we may find that neither (43) nor (45) is rejected and yet (44) is. This would leave us with the familiar situation when faced with nonnested alternatives of rejecting neither alternative.

Before we can estimate our system, we need to specify the stochastic properties of the error term in (39) and to take account of the fact that the index $b(\mathbf{p}^t)/b(\mathbf{p}^{t+1})$ is not known at time t. We shall assume that the error term is white noise and is uncorrelated with all current information except y_t. The possibility that total expenditure might be endogenous has been much discussed in the demand study literature (see Deaton [1983] for a discussion and references). In the context of our model, there are at least two reasons why we might want to instrument total expenditure. First, it depends explicitly on demands (see the derivation of corollary 2). Second, it is measured with error to the extent that the price index used to deflate expenditure in y is not equal to $a(\mathbf{p})$. As instruments we use (log) current disposable income and lagged ln y as well as current and lagged (log) prices.

Now we need to deal with the fact that the lead price index is not observed at time t. To do this we shall employ a familiar rational expectations technique (see, e.g., Cumby, Huizinga, and Obstfeld 1983). We replace $b(\mathbf{p}^t)/b(\mathbf{p}^{t+1})$ by its expectation and then assume that the forecast error is orthogonal to all information at time t. This gives (39) with the realized value of $b(\mathbf{p}^t)/b(\mathbf{p}^{t+1})$ and a composite error term that is, by construction, correlated with this future price index. Also, this composite error may have an MA(1) structure if surprises in the lead index are significant as compared with the white-noise error term e_{it} in (39).[10] Noting that the lead price index depends only on relative prices that change systematically over time, we use as instruments a trend, the current and lagged levels of prices, and the change in (log) import prices. Thus our full instrument set is current and lagged (absolute, log) prices, (log) current disposable income, lagged (log) deflated total expenditure, the first difference in (log) import prices, and a trend.

We shall proceed as follows. We can estimate (39) with (40), (41), and (43) imposed; with these restrictions the system estimates depend only on relative prices and are hence invariant to the discount factor used to discount prices and total expenditure. Since there is no consensus on the proper nominal rate to use, when we are approximating a world with many assets by one with a single asset, there is an obvious gain in robustness. We shall not take account of possible autocorrelation (but shall test for it) and shall assume that the forecast error is conditionally homoscedastic so that we can estimate our system by nonlinear three-stage least squares. Additionally, with these restrictions, the system automatically satisfies adding-up so that we need to drop one equation from the system; we shall drop the services equation.

We have two sets of specification tests for these estimates. The first are "internal" tests for autocorrelation, misspecification in the instruments, and homogeneity; these are given in table 3. The Breusch-Godfrey (1981) statistic is the Lagrange multiplier test statistic for first-order autocorrelation. Under the null hypothesis of no MA(1), these statistics are distributed as a standard normal. As can be seen, there is no evidence of any significant first-order autocorrelation in any of the equations. In what follows we shall not allow for any MA(1) structure; this is consistent with the forecast errors on the future index being small relative to the other errors in the system.[11]

[10] The explicit derivations here are standard and are not given for the sake of presentational parsimony.

[11] In an earlier, four-good version of this system, I did estimate allowing for an MA(1) error using conventional Hansen (1982) methods. I found that the estimates allowing for an MA(1) were very similar to those with no allowance made. This is not surprising given the values of the Breusch-Godfrey statistics found.

TABLE 3

Diagnostics for the SNAP System

Good	Breusch-Godfrey Statistic for MA(1)	Sargan Statistic*	t-Value for Homogeneity
Durables	.46	6.40 (17%)	−.17
Food	−.46	6.01 (20%)	−1.15
Alcohol	−.21	7.40 (12%)	.18
Clothing	−.23	3.59 (46%)	−.43
Fuel	.52	9.64 (4.7%)	−.08
Other goods	.10	5.12 (28%)	−.98

Note.—Figures in parentheses are probabilities under the null.
* Four degrees of freedom.

The next column of table 3 presents the Sargan statistics for the overidentifying restrictions (see Godfrey 1988). Since there are four more instruments than endogenous variables, this statistic has a $\chi^2(4)$ distribution under the null hypothesis that the instruments are orthogonal to the error term. Essentially this tests whether the instruments should be on the right-hand side of the system. As can be seen, only the fuel equation seems to have a problem; we shall ignore this in what follows.

The final column of table 3 records the t-statistic on the (log) price of services if it is included as a regressor in each equation of the system. This is equivalent to relaxing (40) above. As can be seen, current homogeneity is not rejected for any good.[12] These results suggest that allowing for a SNAP structure "solves" the problems displayed by the additive model.

Table 4 presents some "external" specification checks for the SNAP system. For these the quasi likelihood ratio test of Gallant and Jorgenson (1979) is used. In the first set of tests (less restricted variants) we relax, in turn, (40), (41), (40) and (41), and (43). All these relaxations are rejected; that is, we do not reject (40), (41), and (43). The nonrejection of (40) simply confirms the inference drawn from column 3 of table 3. For the last of these tests we estimate the system without (43). This allows us to test for myopic behavior; in fact we find that

[12] The specific values for these test statistics depend on the rate of interest used to discount prices. The use of the absolute prices is equivalent to assuming a constant nominal rate. Experiments with other rates (the 90-day Treasury bill rate and the consol rate) gave the same qualitative results. This is true for all the tests below in which the rate of interest used to discount prices and expenditures matters.

TABLE 4

VARIANTS OF THE SNAP SYSTEM

Model	Degrees of Freedom	χ^2 Statistic
	Less Restricted Variants	
No current homogeneity	6	7.0 (32%)
No lagged homogeneity	1	2.7 (10%)
No homogeneity	7	8.2 (32%)
Unrestricted (but homogeneous)	6	5.8 (45%)
	Restricted Variants	
Symmetry	39	65.9 (.5%)
No SNAP	6	60.6 (0%)

none of the coefficients on the lead index has a *t*-value greater than one (in absolute value). On the other hand, (43) is not rejected either. Thus these data do not allow us to discriminate between myopic and rational habit formation.

When we come to testing more restricted versions of our system, we see that symmetry is rejected and so is the hypothesis that preferences are intertemporally additive. To give some idea of which goods have significant intertemporal dependencies, table 5 presents some parameter estimates for the system with (40), (41), and (43) imposed. It also presents the estimates of β from the system with no SNAP (these are with the same instruments as those in the SNAP system). The estimates for the services equation are derived from the adding-up restriction.

There are two features of table 5 that should be noted. First, there are large differences between the values for β from the system that allows for SNAP and those from the system that does not. I shall return to this below when I discuss elasticities, but at present I shall simply note that as the lagged price index enters (39) as a "deflator" for current total real expenditure, it is not surprising from a statistical point of view that it affects so strongly the coefficient on this variable.

The second thing to note about the estimates in table 5 is that for most goods the product of β and θ is insignificant.[13] Referring back

[13] Since the $\beta\theta$'s are not estimated directly, we must resort to a linear approximation to find the standard error. In this and any following case, for any function $f(\mathbf{k})$ of estimated parameters $\mathbf{k}$ that have covariance matrix $\mathbf{\Omega}$, we estimate the variance of f to be $\mathbf{g}'\mathbf{\Omega}\mathbf{g}$, where $\mathbf{g}$ is the gradient of f.

TABLE 5

SOME PARAMETER ESTIMATES

	No SNAP, β_i	SNAP System β_i	α_i	μ_{ii}	θ_i	$\beta_i\theta_i$
Durables	.62	8.81	11.1	4.06	102.3	9.01
	(5.6)	(4.1)	(66)	(3.2)	(28.5)	(3.89)
Food	−12.7	−1.19	−6.2	15.2	−38.7	.46
	(3.6)	(2.5)	(33)	(4.5)	(22.2)	(1.07)
Alcohol	2.26	−.77	42.4	4.29	21.1	−.16
	(2.3)	(1.2)	(13.6)	(1.1)	(13.0)	(.22)
Clothing	3.77	−2.20	16.3	4.25	−17.7	.39
	(2.2)	(1.7)	(22.6)	(1.1)	(14.5)	(.49)
Fuel	2.80	−3.06	67.6	1.91	22.8	−.70
	(2.9)	(1.3)	(16.7)	(1.4)	(11.6)	(.40)
Other goods	4.49	−5.15	70.7	5.36	−1.7	.09
	(1.3)	(1.7)	(22.9)	(1.2)	(12.3)	(.64)
Services	−1.24	3.56	−102	−1.86	−88.1	−3.14

NOTE.—Standard errors are given in parentheses. All estimates and standard errors are multiplied by 100.

to the discussion following (37), we see that this implies that most goods are autoindependent. Indeed only durables and (to a lesser extent) fuel display any significant auto effect. It is encouraging that durables are highly autosubstitutable. Fuel, on the other hand, looks as though it might be autocomplementary (i.e., habit-forming), but given the results on the Sargan test for this good, I would not want to push this too far. Although alcohol and clothing have the expected signs, they are not significant.

Table 6 presents some of the elasticities implied by the SNAP system estimates. Columns 1–4 give the responses to a permanent price shift for the particular good in period t; see (35)–(37). Column 1 gives the effect of a change in price with everything else held constant (i.e., a Marshallian response). Generally, however, an anticipated change in relative prices also leads to a change in the allocation of expenditure between periods. Column 2 gives the price response if the intertemporal substitution elasticity is minus one. Only for durables is the difference of any magnitude; for all other goods the distinction between the two price elasticities is of little importance. Column 3 of table 6 gives the changes in demand in the periods preceding and following the price change; see (35) and (37). To evaluate these, set $w_i^{t-1} = w_i^t = w_i^{t+1}$ and $b(\mathbf{p}^{t-1}) = b(\mathbf{p}^t)$; this makes only insignificant differences in the estimated elasticities. As one would expect from table 5 and (35) and (37), only durables and fuel show any such effect.

The final price elasticity in table 6 is the total Frisch effect; this is just the sum of column 2 and twice column 3. This can be thought

TABLE 6

PRICE AND EXPENDITURE ELASTICITIES

	Price Elasticities				
	$t-1$ to t				
Good	Marshallian (1)	Frisch (2)	$t-2$ to $t-1$ and t to $t+1$ (3)	Total Frisch (4)	Expenditure Elasticity (5)
Durables	−1.54	−1.69	.81	−.07	1.79
	(.78)	(.71)	(.35)	(1.03)	(.37)
Food	−.22	−.21	.02	−.16	.94
	(.22)	(.21)	(.06)	(.21)	(.13)
Alcohol	−.61	−.60	−.01	−.63	.94
	(.08)	(.09)	(.02)	(.08)	(.10)
Clothing	−.48	−.46	.05	−.36	.73
	(.18)	(.18)	(.06)	(.21)	(.21)
Fuel	−.48	−.46	−.08	−.62	.66
	(.26)	(.26)	(.05)	(.24)	(.15)
Services	−.83	−.87	−.11	−1.09	1.13
	(.18)	(.16)	(.08)	(.10)	(.10)

NOTE.—Standard errors are given in parentheses. 1980 prices and budget shares are used in the evaluation of elasticities and standard errors.

of as the long-run effect of a price change; for an anticipated change it should be negative. Alcohol, fuel, clothing, and services all display a negative long-run price response, and these are not very different from the short-run response (i.e., the goods are autoindependent). Perhaps the most interesting effect, however, pertains to durables: although the short-run effects are large and significant, they offset each other so that the long-run effect of a change in durables prices on durables purchases is small and insignificant.

Column 5 of table 6 gives the expenditure elasticities computed from (38). Food, alcohol, and clothing have elasticities that are not significantly different from unity.[14] On the basis of these estimates, we would categorize durables as a luxury and fuel and other goods as necessities. This categorization is very different from that implied by the estimates that ignore intertemporal dependencies (compare cols. 1 and 2 of table 5).

VI. Conclusions

That preferences over purchases are not separable over time (even if one takes a time period such as a year) seems to be well established.

[14] This finding of a unit elasticity for food on aggregate data is not necessarily at variance with Engel's law, which has been confirmed in numerous cross-section studies (see Deaton [1985] for a discussion of the apparent contradiction between Engel's law and the long-run unit elasticity displayed by aggregate data).

What is a good deal less obvious is the source of such intertemporal dependencies. In this paper I have presented a SNAP structure for preferences that gives closed forms for demands that depend only on current variables and lead and lagged prices. The principal reservation I have about this formulation is that one needs to assume that agents have point expectations about next-period (discounted) prices. I have suggested a small number of strategies for estimating the parameters of such preferences and have implemented what seems to me to be the most robust of them on U.K. aggregate time-series data for seven goods.

The system allowing for a SNAP structure seems to do a lot better at characterizing the aggregate data than the time-separable model does. However, there are still some outstanding problems. The principal of these are that there seems to be more going on in the fuel equation than is adequately captured by the model (see table 3) and that symmetry is rejected. Conditional on my reservations about these, I present the following conclusions: (1) Time separability is rejected. (2) Once one allows for intertemporal nonseparabilities, neither the rational nor the myopic variants of the model are rejected by the data. (3) Not accounting for intertemporal dependencies biases considerably the estimates of intratemporal allocation. For example, the categorization of goods as luxuries or necessities is considerably changed by allowing for nonseparabilities over time. (4) The nonseparability over time is "concentrated" on durables and fuel. All other goods are autoindependent. This does not, of course, mean that one can model them in a time-separable system that leaves out durables and fuel. The results above strongly suggest that doing this would lead to apparent autodependencies in these goods that are the result of incorrectly excluding the goods that do display some autodependence. As seen above, simply including durables in the time-separable model with no allowance for durability improves things somewhat (see table 2), which suggests that leaving out durables in previous models has led to bias and dynamic misspecification. (5) For durables, there is no long-run price effect, but there are significant short-run effects. The pattern observed is that purchases fall in the period in which the price rises but rise in the preceding and following periods even though the price stays at the new level. Thus anticipated changes in the price of durables affect the timing of purchases but not their long-run level. (6) There is no evidence that alcoholic beverages and tobacco as a composite are significantly habit-forming when one takes a year as the consumption period. It is also the case that this composite is price elastic and has unitary expenditure elasticity. (7) There is some evidence that fuel is habit-forming. That is, agents reduce purchases of fuel in the period preceding a price rise as well as in the period of higher prices itself. It may be, however, that this effect is spurious given that there is some evidence that the fuel equation is misspecified.

As discussed at the end of Section III, I chose here to model a conditional (on total expenditure) system. There is, however, no conceptual problem with estimating a full intertemporal model that has a SNAP structure on the data used here. This effectively involves estimating a consumption function jointly with the demand system. Further discussion of this in an intertemporally additive context can be found in MaCurdy (1983), Anderson and Browning (1989), and Blundell, Browning, and Meghir (1990).

The use of a SNAP system is not limited to aggregate time-series demand data. Indeed it seems ideal for use on micro data on demands. The problem with estimating intertemporally nonadditive models on time series of surveys of family expenditures (such as the U.K. or the Canadian Family Expenditure Survey) is that families are surveyed only once, so that one cannot observe lagged purchases. One does, however, observe lagged (and lead) prices for each household since this depends only on the sample period. It is also possible to employ a SNAP structure on models of labor supply on panel data since lagged and lead wages are observed in such data.

Appendix A

The Pollak-Spinnewyn Model

The most general form of the Pollak-Spinnewyn model is

$$\mathbf{s}^t = \mathbf{M}\mathbf{q}^{t-1} + (\mathbf{I} - \mathbf{M})\mathbf{s}^{t-1}, \tag{A1}$$

where $\mathbf{q}^{t-1}$ are purchases in period $t - 1$, $\mathbf{s}^t$ is a vector of "stocks" of the n goods, and $\mathbf{M}$ is a diagonal memory matrix. Preferences are defined over adjusted quantities:

$$\overline{\mathbf{q}}^t = \mathbf{q}^t + \mathbf{H}\mathbf{s}^t, \tag{A2}$$

where $\mathbf{H}$ is a diagonal "habits" matrix. The budgets in each period are linked by the asset evolution equation

$$A_{t+1} = A_t - \mathbf{p}^t\mathbf{q}^t, \tag{A3}$$

where everything is in discounted terms. If preferences over $(\overline{\mathbf{q}}^1, \ldots, \overline{\mathbf{q}}^T)$ are intertemporally additive, then we shall have a standard problem if we can define adjusted prices $(\overline{\mathbf{p}}^t)$ and adjusted wealth $(\overline{A}_t)$ so that we can write our asset evolution equations as

$$\overline{A}_{t+1} = \overline{A}_t - \overline{\mathbf{p}}^t\overline{\mathbf{q}}^t. \tag{A4}$$

To do this, premultiply the one-period-ahead version of (A1) by an arbitrary vector $\boldsymbol{\mu}^{t+1}$ and take this from (A3), to define $\overline{\mathbf{p}}^t$:

$$\begin{aligned} A_{t+1} - \boldsymbol{\mu}^{t+1}\mathbf{s}^{t+1} &= A^t - \mathbf{p}^t\mathbf{q}^t - \boldsymbol{\mu}^{t+1}\mathbf{M}\mathbf{q}^t - \boldsymbol{\mu}^{t+1}(\mathbf{I} - \mathbf{M})\mathbf{s}^t \\ &= A^t - \overline{\mathbf{p}}^t\mathbf{q}^t - \boldsymbol{\mu}^{t+1}(\mathbf{I} - \mathbf{M})\mathbf{s}^t \\ &= A^t - \overline{\mathbf{p}}^t\overline{\mathbf{q}}^t - [\boldsymbol{\mu}^{t+1}(\mathbf{I} - \mathbf{M}) - \overline{\mathbf{p}}^t\mathbf{H}]\mathbf{s}^t \end{aligned} \tag{A5}$$

from (A2). Now define $\overline{A}_t = A_t - \boldsymbol{\mu}^t\mathbf{s}^t$ so that

$$\overline{A}_{t+1} = \overline{A}_t - \overline{\mathbf{p}}^t\overline{\mathbf{q}}^t - [\boldsymbol{\mu}^{t+1}(\mathbf{I} - \mathbf{M}) - \overline{\mathbf{p}}^t\mathbf{H} - \boldsymbol{\mu}^t]\mathbf{s}^t. \tag{A6}$$

Now define $\boldsymbol{\mu}^t$ recursively by fixing some future $\boldsymbol{\mu}^{t+s}$ and letting

$$\boldsymbol{\mu}^t = \boldsymbol{\mu}^{t+1}(\mathbf{I} - \mathbf{M}) - \overline{\mathbf{p}}^t\mathbf{H}. \tag{A7}$$

This gives $\overline{A}_{t+1} = \overline{A}_t - \overline{\mathbf{p}}^t\overline{\mathbf{q}}^t$, where

$$\overline{\mathbf{q}}^t = \mathbf{q}^t + \mathbf{H}[\mathbf{M}\mathbf{q}^{t-1} + (\mathbf{I} - \mathbf{M})\mathbf{M}\mathbf{q}^{t-2}\ldots] \quad \text{from (A1) and (A2)}$$

and

$$\begin{aligned}\overline{\mathbf{p}}^t &= \mathbf{p}^t + \boldsymbol{\mu}^{t+1}\mathbf{M} \quad \text{from (A5)}\\ &= \mathbf{p}^t - \mathbf{p}^{t+1}\mathbf{H}\mathbf{M} - \mathbf{p}^{t+2}\mathbf{H}(\mathbf{I} - \mathbf{M} - \mathbf{M}\mathbf{H})\mathbf{M}\ldots \quad \text{from (A7).}\end{aligned}$$

Thus $\overline{\mathbf{q}}^t$ depends on all past q's and $\overline{\mathbf{p}}^t$ depends on all future prices.

The two special cases mentioned in the text take $\mathbf{H} = (\mathbf{I} - \mathbf{M})\mathbf{M}^{-1}$ (the HAD model) or $\mathbf{M} = \mathbf{I}$ (the short-memory model). It is simple to check that these lead to simple forms for $\overline{\mathbf{p}}^t$ and $\overline{\mathbf{q}}^t$, respectively, but not for $\overline{\mathbf{q}}^t$ and $\overline{\mathbf{p}}^t$, respectively. Combining the two gives $\mathbf{H} = 0$, which is the additive model.

Appendix B

Proofs

Proposition 1

Sufficiency is obvious; the following proves necessity.

From (3) and (5), we have

$$\mathbf{q}^t = -\nabla_t\pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r) = g^t(\mathbf{p}^{t-1}, \mathbf{p}^t, \mathbf{p}^{t+1}, r)$$

and

$$\mathbf{q}^{t-1} = -\nabla_{t-1}\pi(\mathbf{p}^1, \ldots, \mathbf{p}^T, r) = g^{t-1}(\mathbf{p}^{t-2}, \mathbf{p}^{t-1}, \mathbf{p}^t, r).$$

By Young's theorem, we have

$$\nabla_{t-1}g^t(\mathbf{p}^{t-1}, \mathbf{p}^t, \mathbf{p}^{t+1}, r) = \nabla_t g^{t-1}(\mathbf{p}^{t-2}, \mathbf{p}^{t-1}, \mathbf{p}^t, r).$$

From this we see that all the third cross-partial derivatives of π with respect to the p_i^t's from different time periods disappear. Thus π is two-additive in $(\mathbf{p}^1, \ldots, \mathbf{p}^T)$ (see Browning 1983); that is, it is the sum of component functions, each of which depends only on prices from any two periods and on r. Furthermore, any $n \times n$ submatrix of the Hessian formed by taking second partials with respect to $\mathbf{p}^t$ and $\mathbf{p}^s$ is zero if $s \neq t - 1$, t, or $t + 1$. Thus the corresponding components in the two-additive representation of $\pi(\cdot, r)$ disappear, and the profit function has the structure given in the proposition.

Proposition 2

For the single-good, three-period case, the Pollak-Spinnewyn model implies

$$u = v_1(q_1) + v_2(q_2 + hmq_1) + v_3[q_3 + hmq_2 + h(1 - m)q_1].$$

Thus we have

$$u_{12} = hm[v_2' + hm(1 - m)v_3''],$$
$$u_{13} = hm(1 - m)v_3'',$$
$$u_{22} = v_2'' + (hm)^2 v_3'',$$
$$u_{23} = hmv_3'',$$

where v_t'' denotes the second derivative of v_t.

A necessary and sufficient condition for SNAP in the three-period case is $u_{12}u_{23} = u_{22}u_{13}$. Combining the Pollak-Spinnewyn model and SNAP gives

$$(hm)^2 v_3''[v_2' + hm(1 - m)v_3''] = hm\{1 - mv_3''[v_2' + (hm)^2 v_3'']\}.$$

Thus we have $hm = 1 - m$, which is the condition for HAD.

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CHAPTER 6

CONCLUSIONS

CONCLUSIONS

The four papers presented in this thesis are all concerned with models of the allocation of work and money over time that can loosely be termed 'life-cycle' models. A common theme that runs through these papers is the idea that life-cycle agents will seek to keep the marginal utility of money constant over time. Along with supplementary assumptions, this provides predictions about both short run (business cycle/high frequency) allocations and long run (life-cycle/low frequency) allocations. For example, one of the best known long run predictions is that for any individual the life-cycle path of consumption will be independent of the life-cycle path of income (subject to the budget constraint and a host of critical assumptions including the absence of complementaries between leisure and consumption and the existence of perfect capital markets). On the short-run side, perhaps the most important prediction of the life-cycle model is that changes in consumption and hours of work should be uncorrelated with past information (once again, subject to a number of assumptions).

This power to bring all intertemporal decisions into a common framework is the major strength of the life-cycle approach. It is also, however, a source of potential weakness: it is entirely plausible that short run and long run plans are made in some entirely different way. For example, in the empirical section of Chapter 2 it was found that although the life-cycle model does a fair job of characterising the life-cycle allocation of hours of work it does not account for the (short run) year to year changes. Despite this, the gain in theoretical coherence from having a common framework within which to analyse many sorts of decisions which have both long run and short run ramifications makes it is is well worth while seeking to 'patch up' the life-cycle framework rather than abandoning it. This is keeping with the my own view that the life-cycle model provides a framework within which to organise our thoughts rather than a definite set of propositions.

The theory of life-cycle allocation using modern theoretical tools is presented in papers Chapters 2 and 4. This analysis allows for several goods, labour supply by different members of the family and the existence of other conditioning variables. The most important insight here is to note that if preferences are additive over time then the marginal utility of money in any period is a 'sufficient' statistic for all extra-period information. Thus it incorporates all of the agent's past experience and all her beliefs about the future in one variable. Although this variable is not observed by the econometrician, under certain conditions it does follow a known stochastic process which allows the investigator various ways to account for the unobservability. Indeed, if there is no uncertainty then the marginal utility of money is a constant and can be dealt with using familiar fixed effect treatments.

Tests of some of the predictions of the simple (additive over time) life-cycle model using both aggregate data and individual household data are presented in Chapters 2 to 4. Very broadly, there are three sets of conclusions from these investigations:

(i) the aggregate time series data we currently have is virtually worthless for testing the theory. Partly this is because important variables are missing but mainly it is because the time series that are available do not have the sort of variability that would be required to discriminate between various theories of intertemporal allocation.

(ii) even the simplest life-cycle model has some explanatory power for the individual household data. Although there are rejections of some of the predictions of the simplest model these are not so serious as to lead me to believe that further investigations along these lines are necessarily doomed to failure.

(iii) many of the failures of the simplest model seem to point in the same direction: behaviour displays too much persistence over time to be consistent with a model that has additive-over-time preferences and budgets. Thus it seems that we need to allow for either some intertemporal complementarities in preferences or some temporal dependence in the budget (or both). One form of the latter is liquidity constraints: if the borrowing rate of interest is higher than the lending rate then past consumption (which partly determines current asset levels) affects the price of future consumption relative to the price of current consumption. Thus the current budget constraint depends on past actions.

Conclusion (iii) above points to the need to develop models of intertemporal allocation that allow for satiation and habits as well as liquidity constraints. This has been the focus of extensive study in the past five years. Chapter 5 contributes to this literature. The main innovation is the introduction of a novel preference structure (SNAP) that is designed for econometric parsimony. This structure avoids the need for various ad hoc devices that past investigators in the field have found expedient. Although this new model has its own drawbacks it is possible to parameterise it and apply it to panel data. In the paper itself the structure is applied to UK aggregate time series data. Given my reservations about using models of individual behaviour on aggregate data I regard these results as mainly illustrative.

All of the analysis in these papers is cast in a conventional neo-classical mode. Households are treated as individuals who have immutable preferences and obey the expected utility axioms. Although this framework is open to question it does have the twin virtues of being tractable and of having enough structure to put constraints on the data. The very simples versions of the life-cycle model do not seem to provide a convincing characterisation of the data. It is still an open question as to whether slightly more general versions are adequate and if so, in which ways we need to relax the simplest models. My own guess is that for the great majority of households in any market economy simple versions of the life-cycle model will provide an adequate description of their behaviour over both the short run and over the long run.

SAMENVATTING

Hoe huishoudens beslissingen nemen over hoeveel er moet worden gespaard en gewerkt staat centraal in veel belangrijke beleidskwesties. De rode draad die door al deze kwesties loopt, is dat individuen en huishoudens beslissingen moeten nemen die gevolgen hebben op de lange en korte termijn. Het meest gebruikte raamwerk waarbinnen economen dergelijke beslissingen analyseren is het levenscyclusmodel. Het uitgangspunt hierbij is dat huishoudens op elk moment in de tijd hun zaken regelen teneinde een intertemporele nutsfunctie te maximaliseren onder de voorwaarde van een intertemporele budgetrestrictie waarbij alle huidige informatie zo efficiënt mogelijk wordt gebruikt. De belangrijkste implicatie van deze vooronderstelling is dat agenten ernaar streven het marginale nut van (verdisconteerde) uitgaven van periode tot periode constant te houden. Dit is een zeer sterk paradigma, aangezien dit, samen met extra veronderstellingen, voorspellingen verschaft over het korte-termijn (hoge frequentie/conjunctuurcyclus) en lange-termijn (lage frequentie/levenscyclus) gedrag van consumptie en arbeidsaanbod (alsmede alle andere levenscyclusbeslissingen zoals scholing, aantal kinderen, beroepskeuze, omvang en samenstelling van beleggingsportefeuilles en pensioen).

Het is cruciaal te beseffen dat het levenscyclusmodel alleen een raamwerk is. Zonder extra structuur aan te brengen, kunnen we geen voorspellingen doen die beperkingen leggen op wat we zouden kunnen waarnemen. Dus alle modellen binnen dit raamwerk (d.w.z. alle levenscyclusmodellen) kunnen worden gekenmerkt door de gebruikte aanvullende veronderstellingen. Voor dit proefschrift is het noodzakelijk om vier sets van extra te gebruiken veronderstellingen te onderscheiden. In het meest eenvoudige geval wordt verondersteld dat agenten "perfect foresight" hebben; preferenties kunnen worden weergegeven door een nutsfunctie die additief is over de tijd en agenten hebben toegang tot perfecte kapitaalmarkten. Hoewel zeer onrealistisch, dient deze versie als een nuttig uitgangspunt. Er zijn verschillende manieren om de veronderstellingen af te zwakken. De minst restrictieve set van veronderstellingen die wordt gebruikt in de artikelen in dit proefschrift laat onzekerheid toe (met verwachte nutsmaximalisatie) en preferenties die niet additief zijn over de tijd.

Het eerste artikel (Hoofdstuk 2, "A Profitable Approach to Labor Supply and Commodity Demands Over the Life-Cycle", met co-auteurs Angus Deaton and Margaret Irish) bevat een nieuwe theoretische analyse en enkele empirische resultaten, gebaseerd op Engelse data. De belangrijkste theoretische vernieuwing van dit artikel is het gebruik van een alternatieve weergave van preferenties in plaats van de directe nutsfunctie. Dit alternatief wordt de winstfunctie benadering genoemd. In de tweede helft van het hoofdstuk worden resultaten gepresenteerd over arbeidsaanbod en consumptie waarbij data wordt gebruikt van het zogeheten Family Expenditure Survey (FES), afkomstig uit het Verenigd Koninkrijk. We implementeren een methode om het gebrek aan panel data op huishoud-niveau te omzeilen, door tijdreeksen van cross-secties te gebruiken om quasi-panel data te construeren. De grondgedachte achter deze procedure is dat, hoewel de steekproef varieert over de tijd, de bevolking dat niet doet (afgezien van sterfte en migratie), zodat de steekproefgemiddelden in elke periode consistente schattingen zijn van het populatiegemiddelde. We tonen vervolgens aan dat de theorie van toepassing is op de populatiegemiddelden.

In het laatste gedeelte van Hoofdstuk 2 worden de theoretische methoden, geïntroduceerd in het eerste gedeelte, gebruikt om een parameterisatie te ontwikkelen zodat het model kan worden geschat door gebruikmaking van quasi-panel data. We passen dit toe op het arbeidsaanbod van mannen en op consumptie. De arbeidsaanbod-data van mannen reproduceren de gestileerde feiten die door anderen zijn gevonden: voor handarbeiders en niet-handarbeiders is er een duidelijke synchronisatie van gewerkte uren en verdisconteerd loon over de levenscyclus. Dit is één van de voorspellingen van het eenvoudige levenscyclusmodel. De daaruit voortvloeiende intertemporele substitutie-elasticiteit is vergelijkbaar met die van studies waarbij gebruik werd gemaakt van Amerikaanse data. Echter, een nadere beschouwing toont enkele problemen. Ten eerste is er een duidelijk bewijs dat het model niet voldoende de veranderingen in het aantal gewerkte uren van jaar tot jaar verklaart. Dus, alhoewel het model het goed lijkt te doen over de levenscyclus, doet het dat niet over de conjunctuurcyclus. Het tweede probleem is dat uit de urenvergelijking lijkt te volgen dat vrije tijd en consumptie complementen zijn, terwijl ze in de consumptievergelijking substituten zijn. Formeel impliceert dit een verwerping van de symmetrievoorwaarde die één van de voornaamste implicaties is van het eenvoudige theorie model.

Gezien de centrale positie in de consumptie- en arbeidsaanbodliteratuur is het eenvoudige levenscyclusmodel vele malen getoetst. In Hoofdstuk 3 ("Eating, Drinking, Smoking and Testing the Life-Cycle Hypothesis") en Hoofdstuk 4 ("A Non-Parametric Test of the Life-Cycle Rational Expectations Model") presenteer ik twee nieuwe verzamelingen toetsen. De eerste is informeler dan de meeste; de tweede is heel wat formeler en vereist niet om de nutsfunctie te specificeren.

In Hoofdstuk 3 is het alternatief voor de levenscyclus hypothese een nogal slecht gedefinieerd allocatiemodel waarin huishoudens hun 'eerste levensbehoeften' in elke periode bevredigen en vervolgens overblijvend huidig inkomen ("supernumerary income") gebruiken voor de aankoop van 'niet-eerste levensbehoeften' en sparen. De argumentatie die gewoonlijk wordt gegeven voor zo'n model is dat huishoudens zich graag zouden gedragen als levenscyclus-hypothese-agenten maar dat zij "liquidity constrained" zijn. Dientengevolge wordt huidige consumptie gelijkgesteld aan huidig inkomen. Een andere argumentatie zou zijn dat huishoudens zich zo gedragen en het levenscyclusmodel simpelweg onjuist is.

Beschouw een huishouden dat van zo'n allocatieprocedure gebruik maakt en waar een kind wordt geboren. Dit heeft twee effecten op inkomen en bestedingen. Ten eerste, één van de ouders verlaat misschien de beroepsbevolking om voor het kind te gaan zorgen. Dit zal leiden tot een daling van het huidige inkomen. Ten tweede, 'behoeften' nemen toe, omdat kinderen onvermijdelijk kosten met zich meebrengen. Deze twee effecten samen veroorzaken een daling van het supernumerair inkomen. De implicatie is dat bestedingen voor goederen, niet behorend tot de eerste levensbehoeften, zoals tabak en alcohol, zullen dalen. Deze conclusie wordt versterkt als we ook aannemen dat de preferenties van de ouders zodanig veranderen dat zij minder gaan drinken en roken ook al kunnen ze zich dat veroorloven. De voorspellingen voor een 'puur levenscyclus' huishouden zijn heel anders. Als agenten vrij kunnen lenen en uitlenen (en preferenties voor tabak en alcohol zijn additief separabel van andere goederen en van kinderen) dan zouden we verwachten dat ouders hun eerdere niveau van alcohol- en tabaksconsumptie handhaven na de geboorte van kinderen. Om dit te doen zouden ze of lenen of minder sparen.

In het tweede gedeelte van dit artikel worden de quasi-panel data, zoals ontwikkeld in Hoofdstuk 2, gebruikt om de toets te implementeren. We vinden dat kinderen geen effect hebben op de tabaksconsumptie en alleen een klein negatief effect op de alcoholconsumptie. Hoewel onder aanzienlijk voorbehoud, lijkt dit enige ondersteuning te geven voor de eenvoudige versie van het levenscyclusmodel, waarin agenten toegang hebben tot een perfecte kapitaalmarkt en de verwachte waarde van een intertemporeel additieve nutsfunctie maximaliseren.

Eén van de zwakheden van alle toetsen van het levenscyclusmodel die in de literatuur gebruikt zijn, is dat zij een bepaalde parameterisatie voor preferenties veronderstellen. In Hoofdstuk 4 ontwikkel ik de niet-parametrische (of "revealed preference") implicaties van het meest eenvoudige ("perfect foresight") levenscyclusmodel. De kenmerkende voorwaarde generaliseert de voorspelling voor het één-consumptiegoed geval: vraag is een negatieve functie van de verdisconteerde prijs. De generalisatie naar verschillende goederen leidt tot een verzameling voorwaarden die kan worden toegepast op elke tijdreeks van aankopen, verdisconteerde prijzen, werkuren en verdisconteerd loon. Het rentepercentage dat wordt gebruikt om prijzen en loon te verdisconteren is natuurlijk de enige rentevoet, die verondersteld wordt te gelden in de perfecte kapitaalmarkt, zoals aangenomen in de veronderstellingen. De verzameling voorwaarden bevat onder andere Varians GARP niet-parametrische voorwaarden. Dit is niet verrassend omdat deze vereist zijn voor 'rationaliteit' binnen de periode terwijl de voorwaarden voor consistentie met het levenscyclusmodel intra- en intertemporele 'rationaliteit' vereisen. Ten slotte wordt aangetoond dat de niet-parametrische voorwaarden perfect aggregeren, d.w.z.,als zij op elke agent van toepassing zijn, dan zijn ze ook van toepassing op de geaggregeerde data. Deze voorwaarden worden toegepast op geaggregeerde Engelse tijdreeksen. Ze worden niet verworpen voor lange sub-periodes. Als we bovendien rekening houden met 'verrassingen' in het begin van de jaren '70 dan zijn de niet-parametrische voorwaarden van toepassing op de gehele periode. Dit is van aanzienlijk belang omdat parametrische toetsen van de levenscyclus hypothese op vergelijkbare data over het algemeen sterk worden verworpen. Het niet-verwerpen van de niet-parametrische voorwaarden duidt erop dat de verwerpingen die in de literatuur zijn beschreven misschien meer een kwestie zijn van het opleggen van ongeschikte parameterisaties dan van het verwerpen van de hypothese op zich.

Hoewel dit er bemoedigend uitziet voor de eenvoudigste vorm van het levenscyclusmodel wijzen de specifieke verwerpingen van de niet-parametrische voorwaarden over de gehele periode in enkele specifieke richtingen. Bijvoorbeeld, in ieder geval waarin de voorwaarden worden verworpen, gebeurt dat omdat consumptie niet zoveel daalt als zou volgen uit de theorie. Dit wijst op een 'ratchet'-consumptiemodel zoals voor het eerst gepresenteerd door Duesenberry aan het eind van de jaren '40. Een andere mogelijkheid is, dat we de periodes waarvoor de niet-parametrische voorwaarden niet gelden, beschouwen als de periodes waarin agenten "liquidity constrained" zijn. Dit alles geeft aan dat het levenscyclusmodel (met enkele gewoonte-vorming of kapitaalmarkt-imperfecties) zeer goed de aggregate tijdreeks-data weergeeft. Wat te denken van het belangrijkste informele alternatief, namelijk een 'Keynesiaans' model waarin we veronderstellen dat huidige totale bestedingen een stijgende functie is van huidig totaal inkomen en dat agenten deze totale bestedingen alloceren gebruikmakend van stabiele preferenties. In de laatste paragraaf van Hoofdstuk 4 wordt aangetoond dat de gebruikte data ook geheel consistent zijn met zo'n hypothese. De slotconclusie is dan dat de aggregate tijdreeks-data eigenlijk waardeloos zijn voor het toetsen van het levenscyclusmodel.

Het laatste artikel in dit proefschrift (Hoofdstuk 5, "A Simple Nonadditive Preference Structure for Models of Household Behavior Over Time") gaat over preferenties die niet additief zijn over tijd. Diverse modellen zijn gesuggereerd voor niet-intertemporele separabiliteit. De benadering in Hoofdstuk 5 is om de eenvoudigst mogelijke generalisatie van additiviteit af te leiden, waar 'eenvoudig' slaat op de databehoeften. Er wordt een preferentiestructuur ontwikkeld, zo dat huidige vraag mag afhangen van prijzen één periode geleden, huidige prijzen en prijzen één periode in de toekomst. Ik karakteriseer de preferenties die aan deze voorwaarde voldoen en duidt deze aan als de "Simple Non-Additive Preference" (SNAP) structuur. In de eerste paragraaf van Hoofdstuk 5 worden enkele van de gevolgen van een SNAP structuur in een wereld met perfecte zekerheid nagegaan. Er wordt aangetoond dat deze structuur het standaard duurzame-consumptiegoederen-geval omvat als een speciaal geval. De tweede paragraaf laat zien hoe het raamwerk moet worden uitgewerkt om met onzekerheid rekening te houden. Het belangrijkste resultaat hier is, wil SNAP empirisch toepasbaar zijn, dan moeten we òf de preferenties in belangrijke mate beperken òf we moeten veronderstellen dat agenten puntverwachtingen hebben aangaande toekomstige prijzen. In de daaropvolgende twee paragrafen wordt het SNAP model toegepast op enkele geaggregeerde Engelse tijdreeks-data. De in dit geval gekozen parameterisatie omvat het "Almost Ideal Demand System" (AIDS) als een speciaal geval. De SNAP generalisatie introduceert alleen enkele parameters meer (om precies te zijn, één minder dan het aantal goederen dat in het model voorkomt). Indien toegepast op Engelse data, blijkt dat intertemporele additiviteit wordt verworpen. Opmerkelijk is echter dat de intertemporele afhankelijkheden grotendeels 'geconcentreerd' zijn op duurzame goederen, wat nauwelijks verrassend is. Eigenlijk lijkt het erop of alle andere goederen (met de mogelijke uitzondering van brandstof die hoe dan ook problemen oplevert) intertemporeel separabel zijn (volgens de definitie van het artikel). Dus het lijkt alsof de meeste van de schijnbare niet-separabiliteiten gevonden in vorige vraagstudies toegeschreven kunnen worden aan het niet opnemen van duurzame goederen in zulke systemen. Omdat de behoeften aan andere goederen niet separabel zijn van duurzame goederen, leidt dit tot een duidelijke behoefte om rekening te houden met intertemporele niet-separabiliteiten in, bijvoorbeeld, voedsel of diensten.